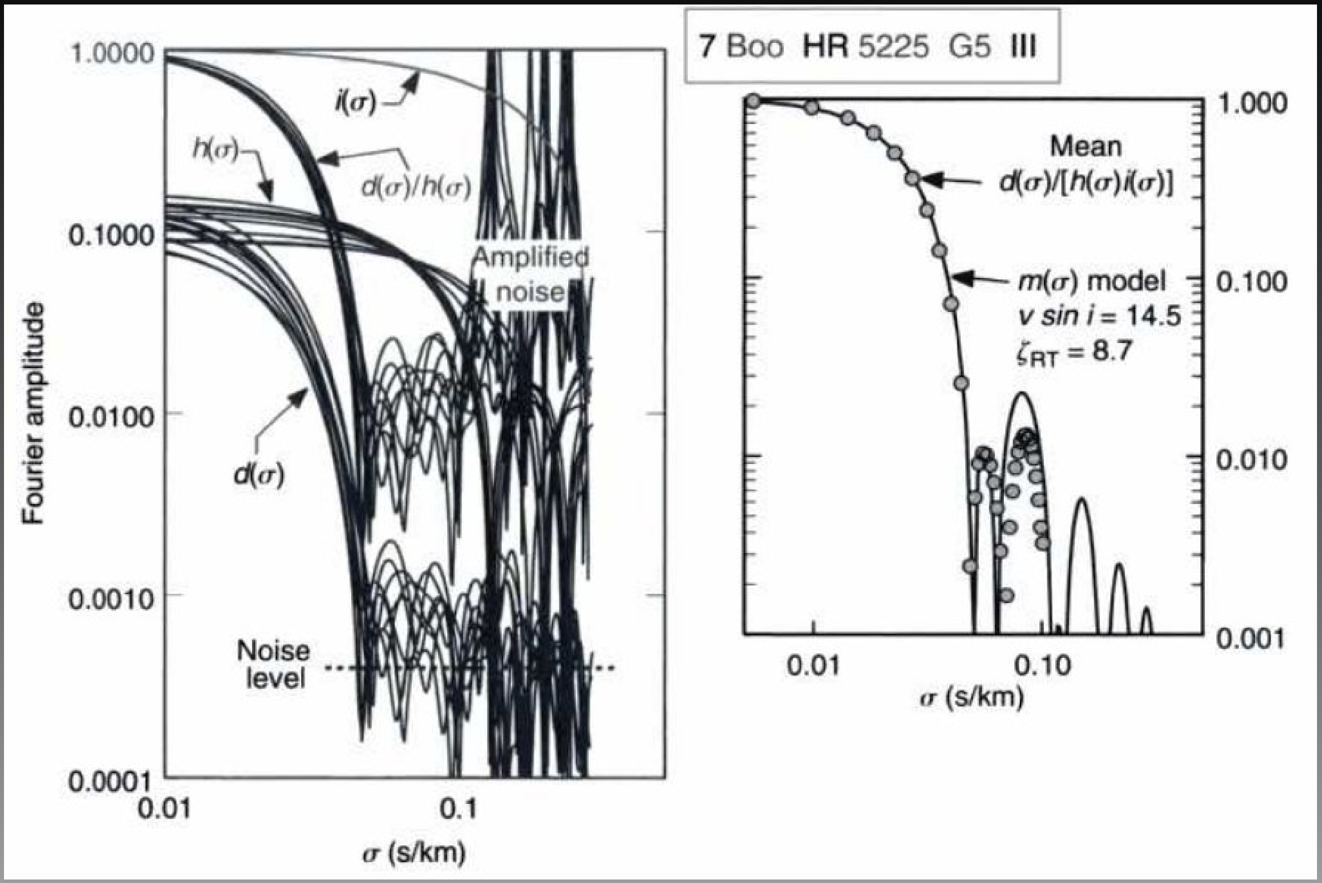
Lecture 20



from Gray 2005

suggested readings:

Lena, Lebrum & Mignard, Observational Astrophysics, pg. 204, 256
Gray, pg. 26

Spectroscopy

sampling & PSF

PSF = I (λ)

$$D(\lambda) =$$

$$F(\lambda)\times W_1(\lambda)\times W_2(\lambda)*I(\lambda)$$

$$\mathscr{F}\left[D(\lambda)\right] =$$

$$d(\sigma)=f(\sigma)*w_1(\sigma)*w_2(\sigma)\times i(\sigma)$$

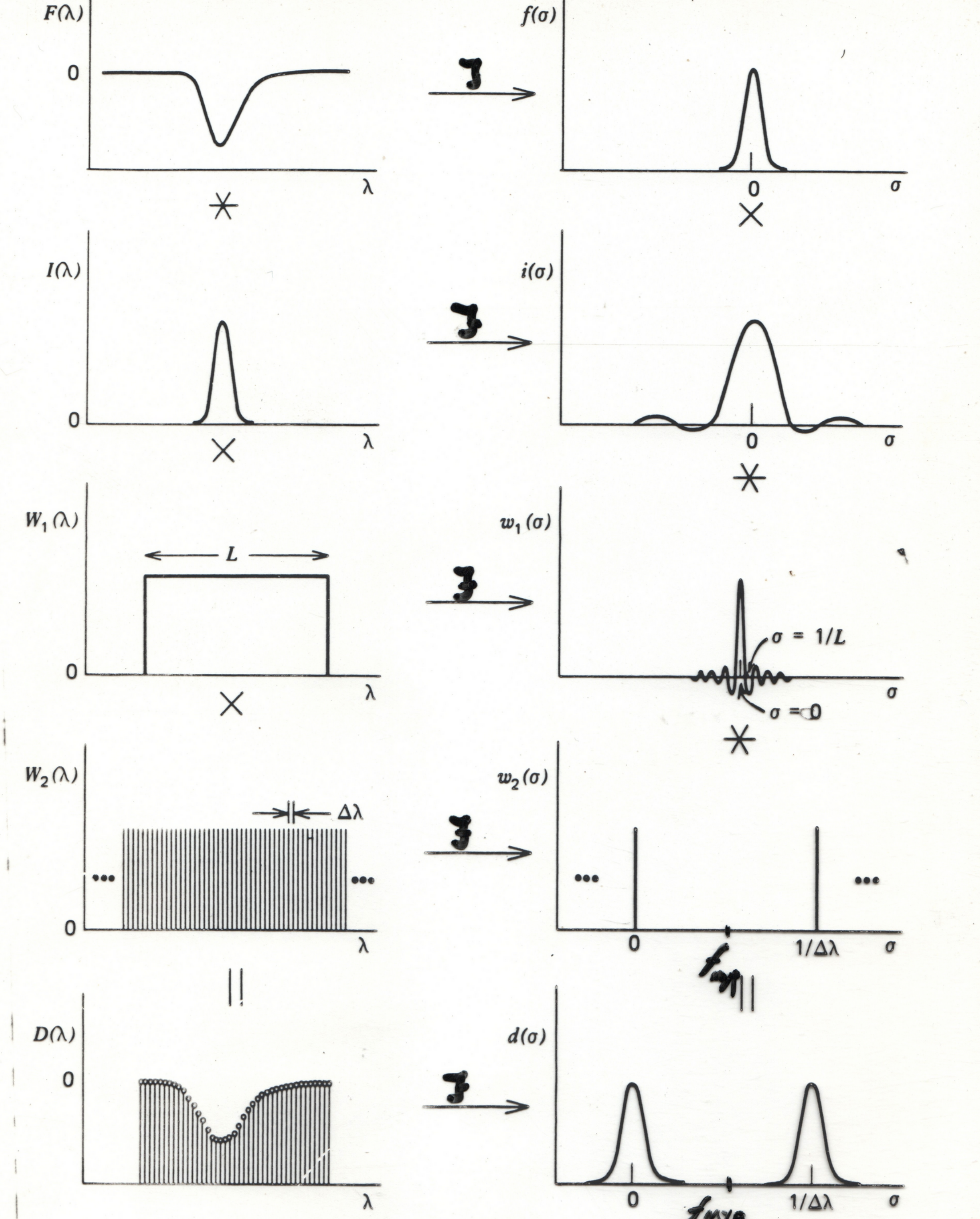
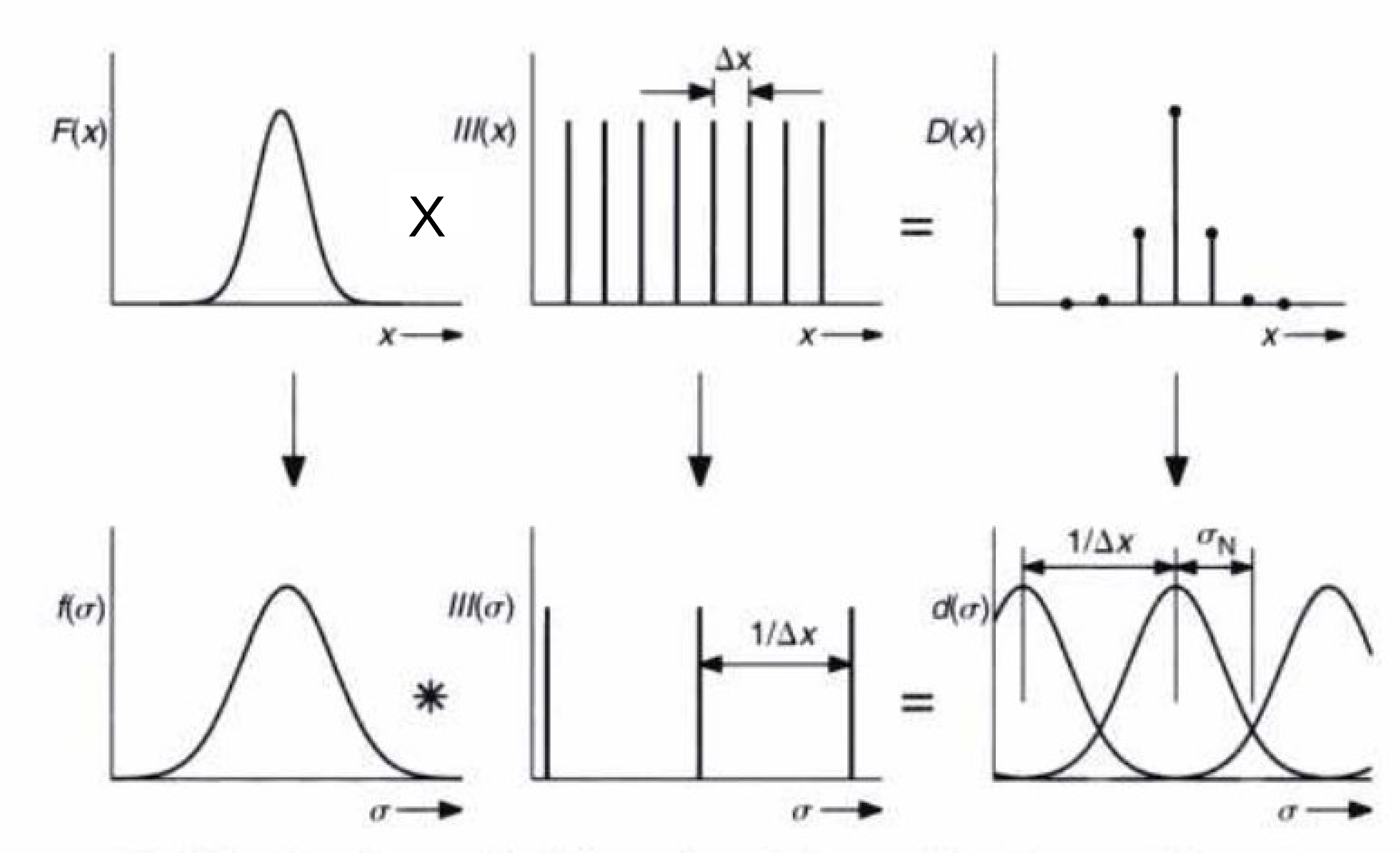


Fig. 12.11. The star's flux profile, which we wish to study, is shown on the upper left. It is blurred by the convolution with the instrumental profile, and then sampled through the windows W_1 and W_2. In the σ domain, the transform of the flux profile on the upper right is filtered by $i(\sigma)$, blurred by w_1, and replicated by w_2 to give the data transform at the lower right. The negative signs on the amplitudes of $D(\lambda)$ and $F(\lambda)$ have been ignored.

Spectroscopy

Aliasing
&
Nyquist limit

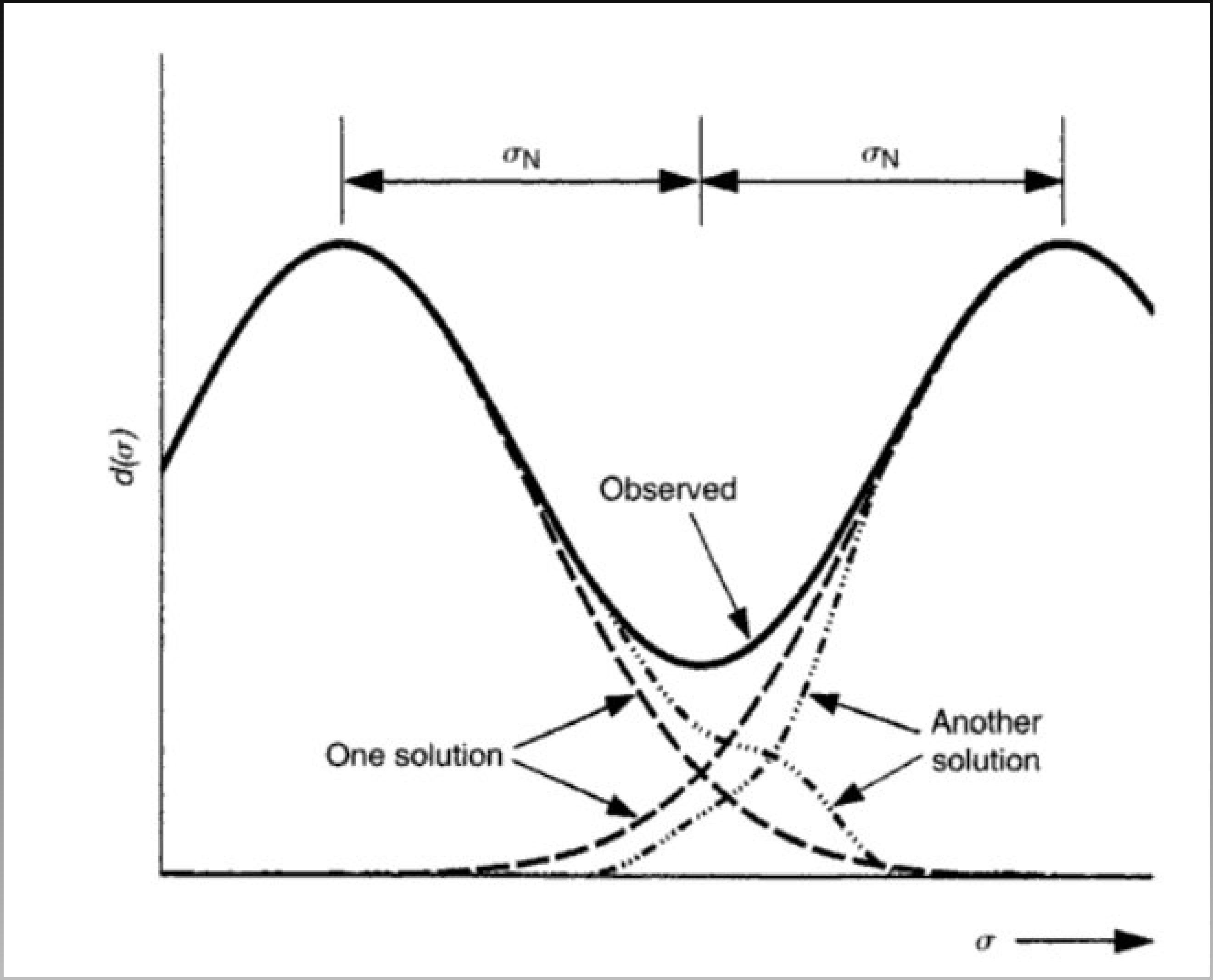


(modified from Gray 2005)

$$\sigma_{Nyq} = \frac{1}{2\Delta\lambda}$$

Spectroscopy

PSF sampling & Nyquist blurring



(from Gray 2005)

$$\sigma_{Nyq} = \frac{1}{2\Delta\lambda}$$

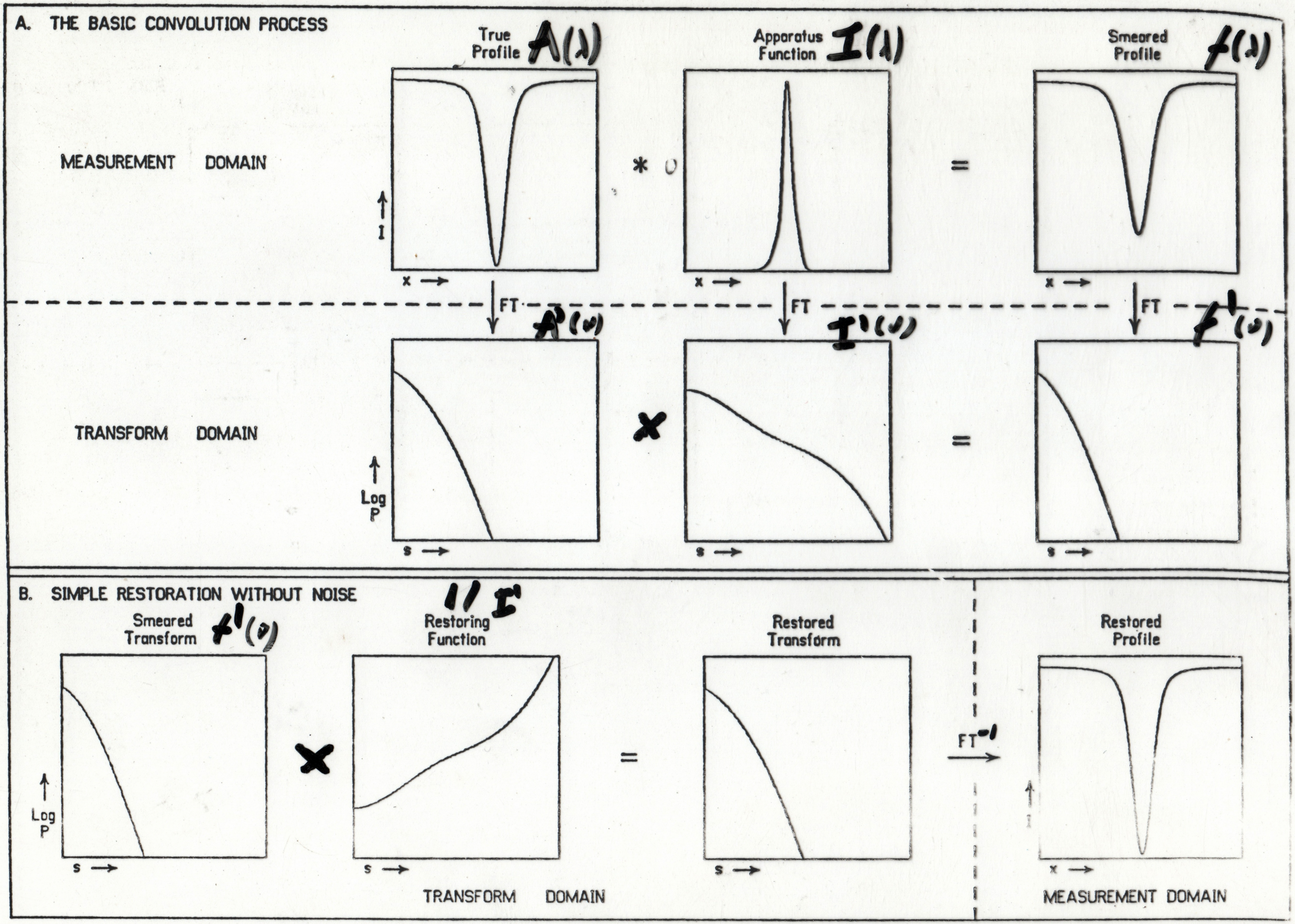


Fig. 12. Illustrations of (A) the convolution process as seen in both domains and (B) the simple Fourier restoration in the absence of noise

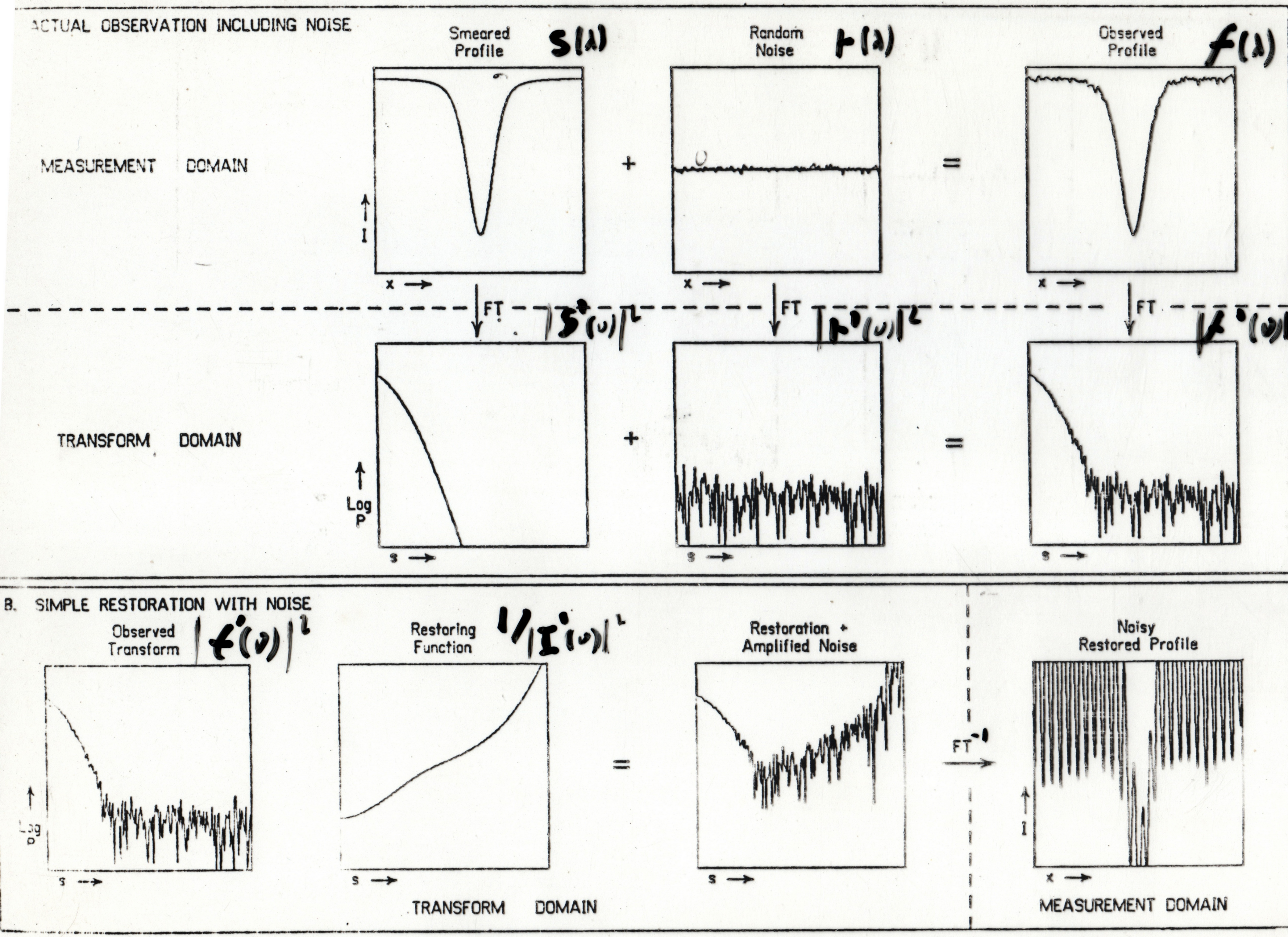


Fig. 13. Illustrations of (A) the effect of noise on the observed profile and its transform, and (B) the disastrous result of noise

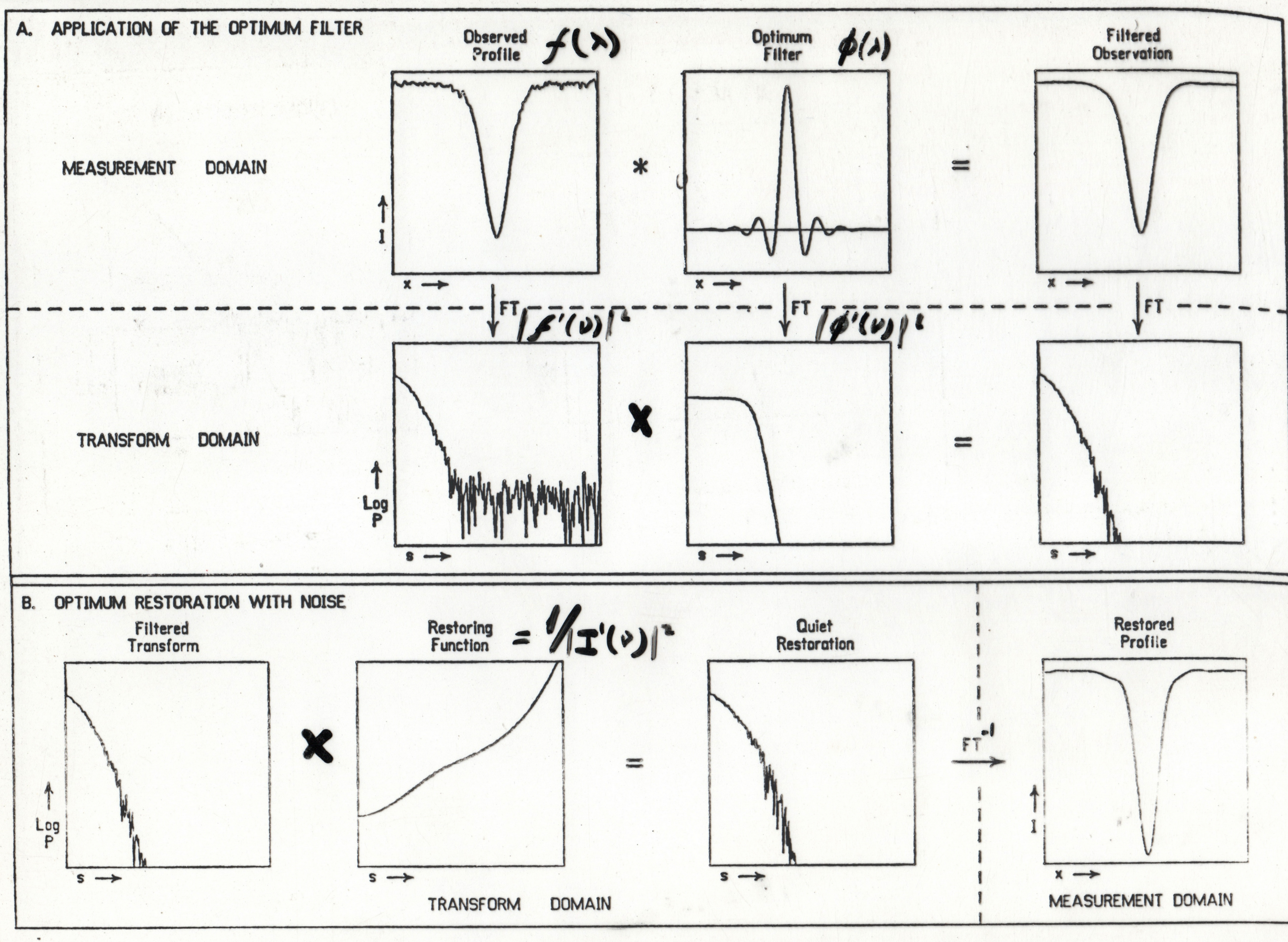


Fig. 14. Illustrations of (A) the noise reduction provided by the optimum filter, and (B) the resulting quiet restoration

The Optimal Wiener filter (ϕ)

$$\tilde{S}'(\nu) = \frac{f'(\nu) \times \phi'(\nu)}{I'(\nu)}$$

filtered estimate of true S' (ν)

that most closely resembles the noiseless spectrum transform S' :

$$\int_{-\infty}^{\infty} |\tilde{S}(\lambda) - S(\lambda)| \, d\lambda = \int_{-\infty}^{\infty} |\tilde{S}'(\nu) - S'(\nu)| \, d\nu$$

$$f'(\nu) = S'(\nu) + r'(\nu)$$

$$\int_{-\infty}^{\infty} \left| \frac{[S'(\nu) + r'(\nu)]\phi'(\nu)}{I'(\nu)} - \frac{S'(\nu)}{I'(\nu)} \right|^2 d\nu$$

all (pure noise) x *(signal) terms integrate to 0*

$$\int_{-\infty}^{\infty} |I'(\nu)|^{-2} \left[|S'(\nu)|^2 |1 - \phi'(\nu)|^2 + |r'(\nu)|^2 |\phi'(\nu)|^2 \right] d\nu$$

The Optimal Wiener filter (cont.)

minimizing variance against ϕ'

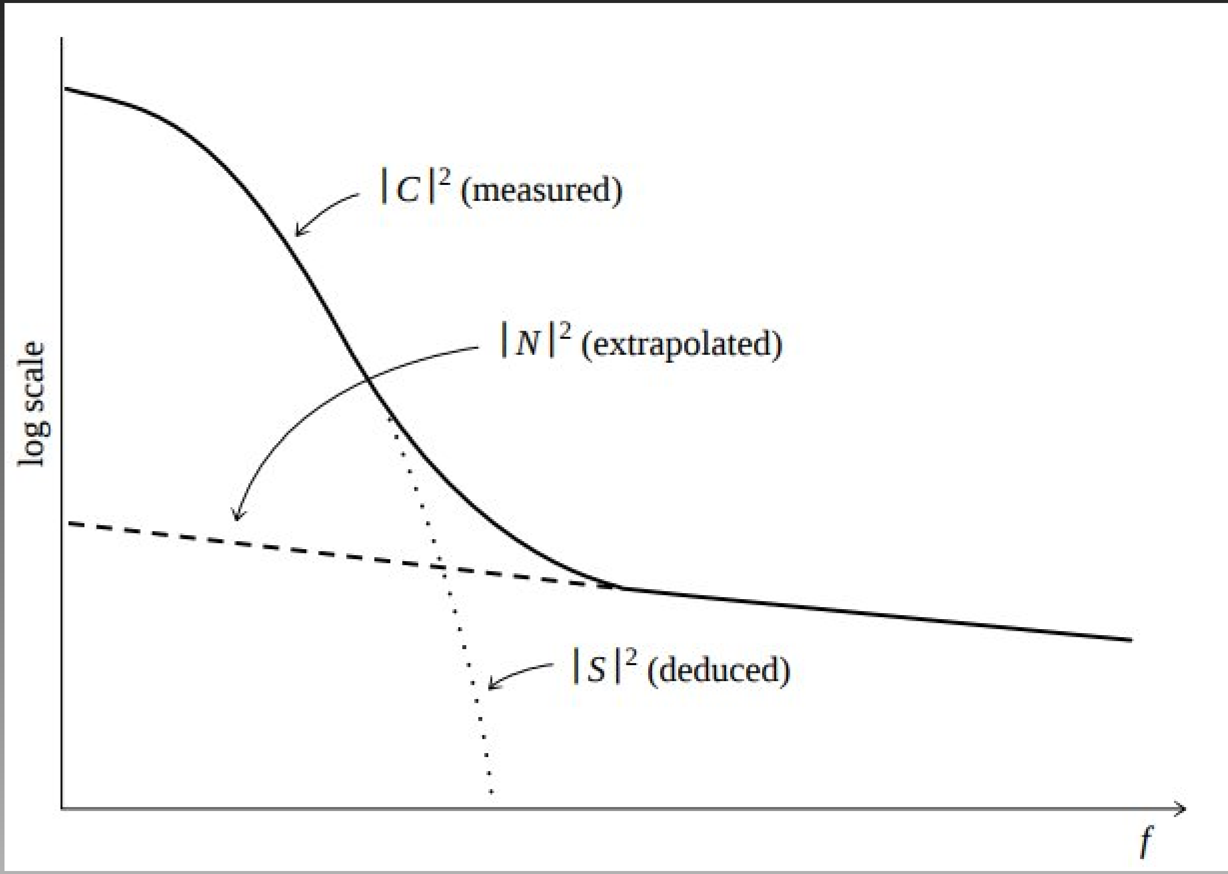
$$\frac{\partial\left[\left|S'(\nu)\right|^2\left|1-\phi'(\nu)\right|^2 + \left|r'(\nu)\right|^2\left|\phi'(\nu)\right|^2\right]}{\partial\phi'} = 0$$

$$\phi'(\nu) = \frac{\left|S'(\nu)\right|^2}{\left|S'(\nu)\right|^2 - \left|r'(\nu)\right|^2}$$

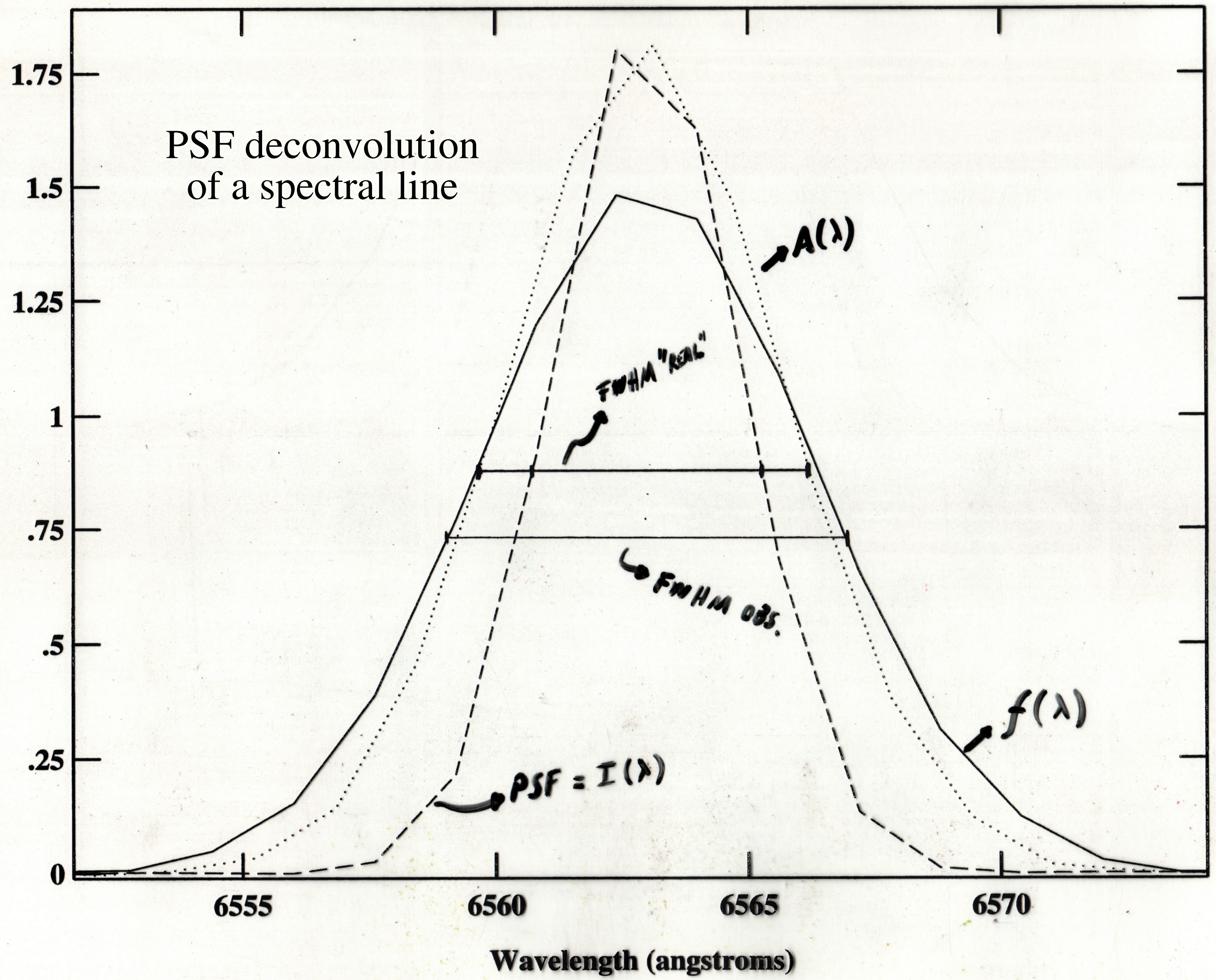
need for composite $\phi'(\nu)$:

- Optimal (Wiener)
  +
- low pass

High (>~ 300) S/N is required to "gain" over instrumental resolution.



Numerical Recipes, Press et al. 1995, pg 547

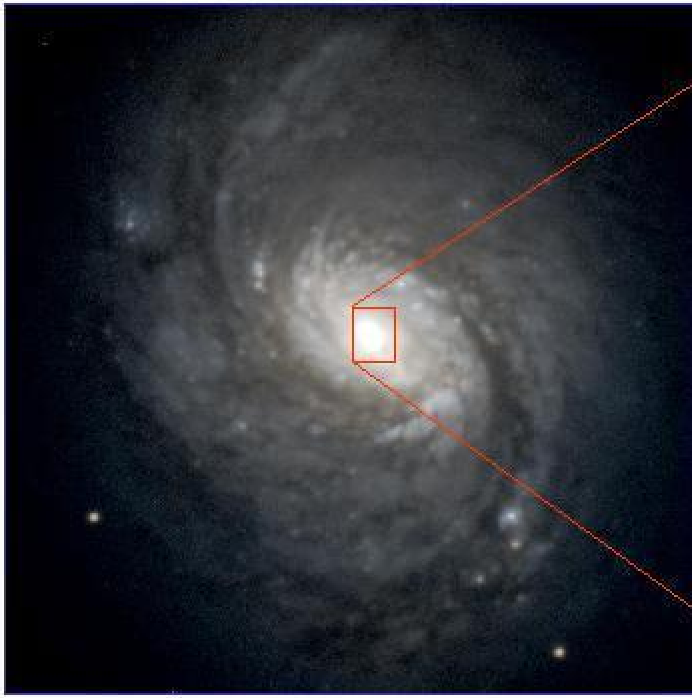


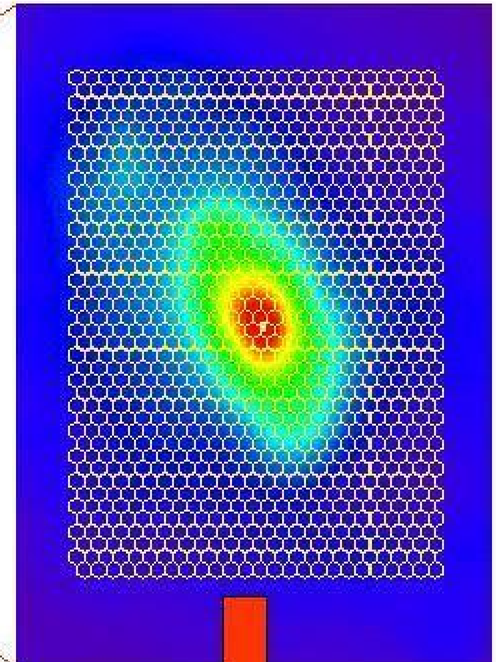
PSF deconvolution
of a spectral line
A(λ)
FWHM "REAL"
FWHM OBS.
f(λ)
PSF = I(λ)
1.75
1.5
1.25
1
.75
.5
.25
0
6555
6560
6565
6570
Wavelength (angstroms)

University of Durham

GMOS Integral Field Unit observes NGC1068

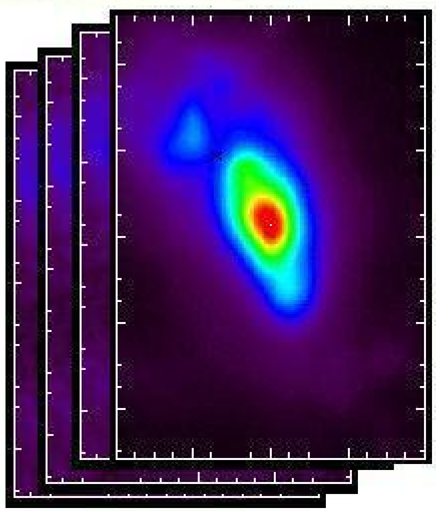
Image taken by GMOS without using the IFU

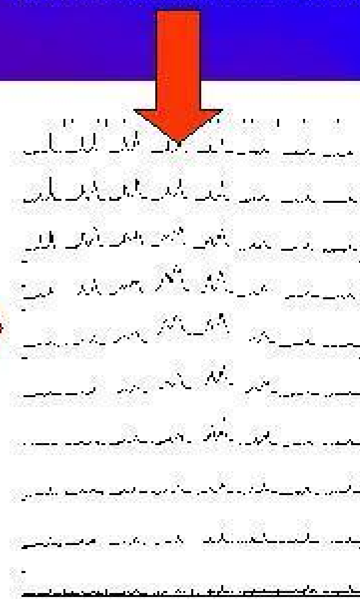




The GMOS IFU records a spectrum for each pixel

One image at each wavelength





One spectrum for each pixel in the image

Optimal Extraction of IFU Stellar Spectra

The minimum variance extraction of a stellar spectrum at a given wavelength is:

$$I(\lambda) = \frac{\sum_{i,j} w(r_i, \theta_j, \lambda) f(r_i, \theta_j, \lambda)}{\sum_{i,j} w(r_i, \theta_j \lambda)}.$$

Here f is proportional to the signal per IFU element.

with

$$w(i, j, \lambda) = \frac{PSF^2(r)}{\sum_l \mathrm{var}[counts(i, j, l, \lambda)]}$$

$$var_{opt}(\lambda) = \frac{1}{\sum_r \left(PSF^2(r) \ / \ var\left[counts(r,\lambda)\right]\right)}$$

$$var_{\text{sum}}(\lambda) = \sum_r var\left[counts(r,\lambda)\right]$$

$$\sum_r PSF(r) = 1$$

(for 1D spectra see Horne, K., PASP,1988)

Case 1 - background limited observations

Now, lets us assume that the background is constant along the PSF and it follows a Poisson statistical distribution. When the variance per pixel is dominated by the sky background we have $\sigma^2 = n_{counts}$ ~ constant (the number of counts per pixel). Summing over the N IFU elements (*i*, *j*) at λ:

$$\frac{Var[I_{sum}]}{Var[I_{opt}]} = N * \sum_{i,j} PSF^2(r)$$ with a gaussian PSF approximation we get:

$$\frac{Var[I_{sum}]}{Var[I_{opt}]} = N * \frac{0.44}{(FWHM)^2}$$

Example: Suppose we integrate over $\pm 3\sigma$ and the stellar profile is sampled by 2.5 IFU elements per FWHM. The corresponding ratio would be 2.3 (in variance) or 1.5 (in RMS).

*** The corresponding ratio for slit spectroscopy is 1.7 in variance or 1.3 in RMS ***

Case 2 - Negligible background and readout noise

This is the case of bright stars (flux standards for example) where source photon statistics is the major contribution to the measurement noise. By assuming Poisson statistics, the variance of each IFU element would be proportional to the local PSF value:

$$\sum_{l} \mathrm{var}[counts(i, j, l, \lambda)] = const * PSF(r_i, \theta_j)$$

applying this in our variance ratio expression (11) and using the normalization condition for the PSF model one find:

$$\frac{Var[I_{sum}]}{Var[I_{opt}]} = 1$$