Lecture 14

Stellar Atmospheres
prof. Marcos Diaz

Harrington, 1970

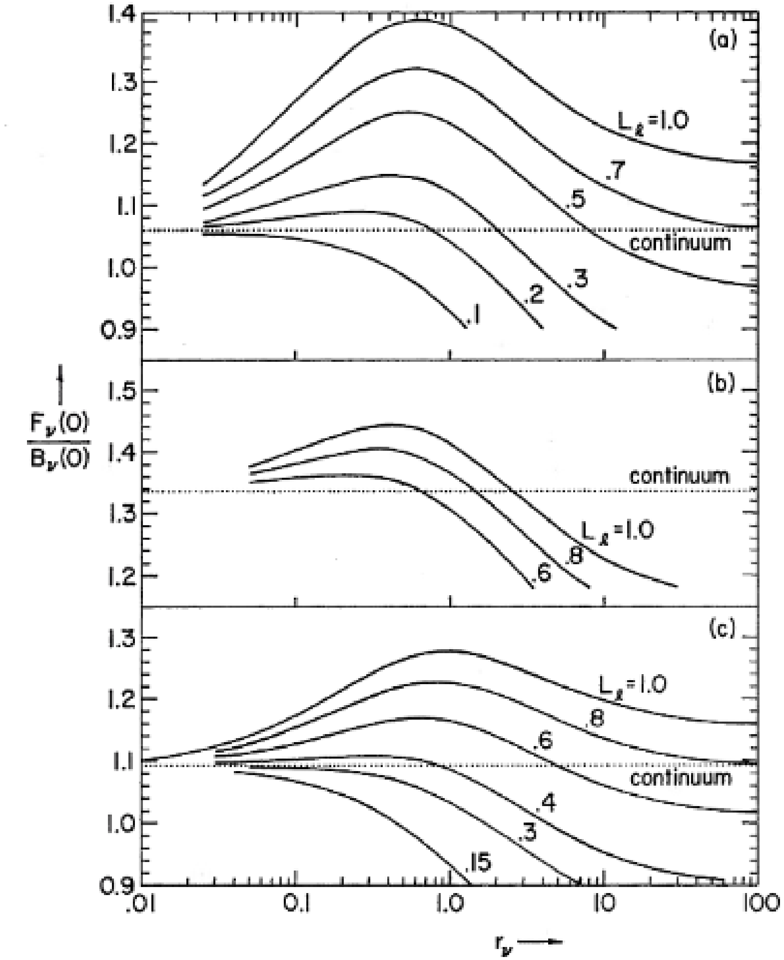

FIG. 2.—Ratios of line flux to the surface value of the Planck function, plotted against the ratio of line extinction to continuous extinction. The cases shown are: (a) $y_e = 2.0$, $L_c = 0.05$; (b) $y_e = 2$, $L_c = 0.15$; (c) $y_e = 1.2$, $L_c = 0.15$. Dotted line is $F_c(0)/B_\nu(0)$.

treasure map:

H&M: pg 607
Mihalas, 1978
Rutten: pg 92
Gebbie & Thomas, ApJ, 154, 285, 1968
Israelian & Nikoghossian, J.Q.S.R.T, 56, 509, 1996

Basic line radiation transfer

line photons are subject to:
- pure absorption (ε)
- scattering (^{s} superscript) (1-ε)
- thermal emission (^{t} superscript)

$$l_L = (1-\epsilon) l_L + \epsilon l_L$$

(line scattering and absorption coef.)

line scattering is:
- isotropic
- complete redistribution ($R^S_{\nu,\phi} = W_\nu$, $1/4\pi$)

$$\frac{\mu}{\rho}\frac{dI_\nu}{dz} = j_L^t + j_L^s + j_C^t + j_C^s - \left[k_C + \sigma_C + l_L\right] I_\nu$$

$$d\tau_\nu = -(k_C + \sigma_C + l_L)\rho\, dz$$

with $$j_L^s = (1-\epsilon) J_\nu l_L$$

(line scattering emission into the beam)

$$\epsilon\, l_L J_\nu = \epsilon\, l_L B_\nu = j_L^t$$ (line thermal emission in LTE)

$$j_C^s = \sigma_C J_\nu$$ (continuum scattering emission)

$$j_C^t = k_C B_\nu$$ (continuum thermal emission in LTE)

$$\frac{\mu}{\rho}\frac{dI_\nu}{dz} = \epsilon\, l_L B_\nu + (1-\epsilon) l_L J_\nu + k_C B_\nu + \sigma_C J_\nu - \left[k_C + \sigma_C + l_L\right] I_\nu$$

with $$\eta_\nu = \frac{l_L}{(k_C+\sigma_C)}$$ (line-to-continuum total abs. ratio)

and $$\zeta_C = \frac{\sigma_C}{(k_C+\sigma_C)}$$ (scattering-to-total abs. ratio for continuum)

$$\mu\frac{dI_\nu}{d\tau_\nu} = I_\nu - \frac{(1-\zeta)+\epsilon\,\eta_\nu}{(1+\eta_\nu)} B_\nu + \frac{\zeta + (1-\epsilon)\eta_\nu}{(1+\eta_\nu)} J_\nu$$

with $$\lambda_\nu = \frac{(1-\zeta)+\epsilon\,\eta_\nu}{(1+\eta_\nu)}$$

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - \lambda_\nu B_\nu - (1-\lambda_\nu) J_\nu$$ *Milne-Eddington equation*

with $$B_\nu(\tau) = B_0 + B_1 \tau$$ and $$B_1 = \frac{C_1}{(1+\eta_\nu)}$$

$$J_\nu(\tau) = B_0 + B_1\tau + \left(\frac{B_1 - \sqrt{3}\,B_0}{\sqrt{3}}\right)\frac{1}{(1+\sqrt{\lambda_\nu})} e^{\sqrt{3\lambda_\nu}\,\tau}$$

with $$J_\nu(0) = \sqrt{3}\,H_\nu(0)$$ (2-stream approx)

$$H_\nu(0) = \frac{1}{3}\left(\frac{B_1 - \sqrt{3\lambda}\,B_0}{(1+\sqrt{\lambda})}\right) \qquad (1)$$

1. continuum

$\eta_\nu = 0; \quad \lambda_\nu = 1 - \zeta$

$$H_{Cont}(0) = \frac{1}{3}\left(\frac{B_1 - \sqrt{3(1-\zeta)}\,B_0}{1+\sqrt{1-\zeta}}\right) \qquad (2)$$

2. lines

$$R_\nu = \frac{H_{Cont} - H_\nu}{H_{Cont}} = \frac{\left(\frac{B_1 - \sqrt{3(1-\zeta)}\,B_0}{1+\sqrt{1-\zeta}}\right) - \left(\frac{B_1 - \sqrt{3\lambda_\nu}\,B_0}{1+\sqrt{3\lambda_\nu}}\right)}{\left(\frac{B_1 - \sqrt{3(1-\zeta)}\,B_0}{1+\sqrt{1-\zeta}}\right)}$$

$$R_\nu = 1 - \frac{\left(C_1/(1+\eta_\nu) + B_0\sqrt{3\lambda_\nu}\right)}{\left(C_1 + B_0\sqrt{3(1-\zeta)}\right)} \cdot \frac{\left(1+\sqrt{1-\zeta}\right)}{\left(1+\sqrt{\lambda_\nu}\right)} \qquad (3)$$

2.i Pure scattering lines & absence of continuum scattering

$\varepsilon = 0; \quad \sigma_C = 0; \qquad \zeta = 0;$

$$\lambda_\nu = \frac{1}{(1+\eta_\nu)}$$

from 1 and 2:

$$\frac{H_\nu}{H_C} = \frac{\frac{2C_1}{(1+\eta_\nu)} + \left(\frac{12}{(1+\eta_\nu)}\right)^{1/2} B_0}{\left(C_1 + \sqrt{3}\,B_0\right)}$$

with $\eta(\nu=\nu_0) = \eta_0 >> C_1$ $\quad \left(\frac{H_\nu}{H_C}\right)_{\eta_0} \rightarrow 0$ (dark core line)

2.ii Pure absorption lines & absence of continuum scattering

$\varepsilon = 1;\quad \sigma_C = 0;\qquad \zeta = 0;$

$$\lambda_\nu = 1 \qquad (achromatic)$$

with $\eta(\nu=\nu_0) = \eta_0 >> C_1$

from 3 ($\nu \approx \nu_0$):

$$R_\nu = 1 - \frac{\sqrt{3}\,B_0}{C_1 + \sqrt{3}\,B_0}$$

$C_1 > 0 \rightarrow \qquad R_\nu > 0$

$$\frac{H_\nu}{H_C} = \frac{\sqrt{3}\,B_0}{\left(C_1 + \sqrt{3}\,B_0\right)}$$

even with $\eta_0 >> C_1$ $\qquad \left(\frac{H_\nu}{H_C}\right)_{\eta_0} \rightarrow \neq 0$ $\qquad$ (non-dark core line)

2.iii Pure absorption line & pure continuum scattering

$\varepsilon = 1$; $k_C = 0$; $\zeta = 1$;

$$\lambda_\nu = \frac{\eta_\nu}{1+\eta_\nu}$$

with $\eta(\nu\sim\nu_0) = \eta_\nu >> C_1$ $\lambda_\nu \rightarrow 1$ *(achromatic)*

from 3 and $\nu\sim\nu_0$

$$R_\nu = 1-\frac{\sqrt{3}}{2}\frac{B_0}{C_1}$$

$$\frac{B_0}{C_1} > \frac{2}{\sqrt{3}} \rightarrow R_\nu < 0$$

$$C_1 = \frac{\partial B_\nu}{\partial \tau}(\tau=0)$$

(line center, more often wings or even whole line in emission for normal temperature gradient)

Schuster mechanism