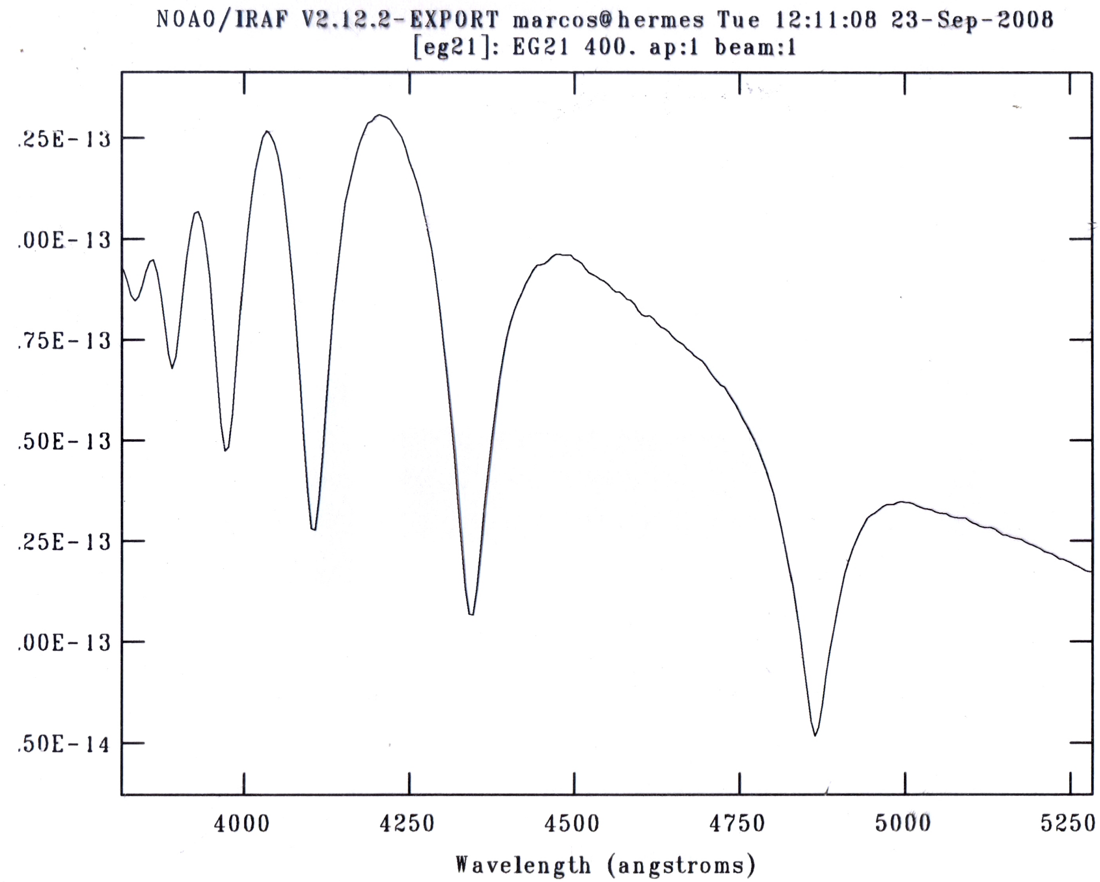
Lecture 10

stellar atmospheres
prof. Marcos Diaz



treasure map:
Gray: pg 247
H&M: pg 233
Bohn-Vitense: pg 136
Rutten: pg 118

Microscopic Line Broadening

III. Semi-classical Stark Broadening

level splitting by external E potential into k sublevels

$$\Delta\omega = C_{k,p}/r^p$$

with p:

p=2: ***linear Stark***. *hydrogenic neutral (dipole) + charged particle or ion (e.g. H and P, e^-, H^-)*
p=3: ***ressonant Stark.*** *same dipole neutral particle (e.g. H and H)*
p=4: ***quadratic Stark.*** *non-hydrog. atom + charged particle or ion (NaI and H^-)*
p=6: ***van der Waals***. *non-hydrog. atom + dipole (e.g. NaI and H)*

quantum Stark theory $C_p \rightarrow C_{k,p}$

III. Semi-classical Stark Broadening *(cont.)*

(a) Impact Lindholm theory vs. (b) quasi-static theory

a. Impact. → shortening of natural decay by fast collisions
nothing between them

add collision probability to natural decay probability:

$$\Gamma = \Gamma_{nat} + \Gamma_{col}$$

Lindholm perturbed frequency:

$$I(\omega) = \frac{(N\overline{v}\sigma_R/\pi)}{(\omega - \omega_0 - N\overline{v}\sigma_I)^2 + (N\overline{v}\sigma_R)^2} \qquad (1)$$

with

N	=	density of (light) perturber
$\sigma_{l,R}$	=	Lindholm, Radiative cross-section
v	=	particle-perturber relative velocity

(1) is a Lorentz profile with: $\Delta\omega = \frac{\Gamma}{2}$

$\Gamma = 2N\overline{v}\sigma_R$ and $\Delta\omega_0 = N\overline{v}\sigma_I$

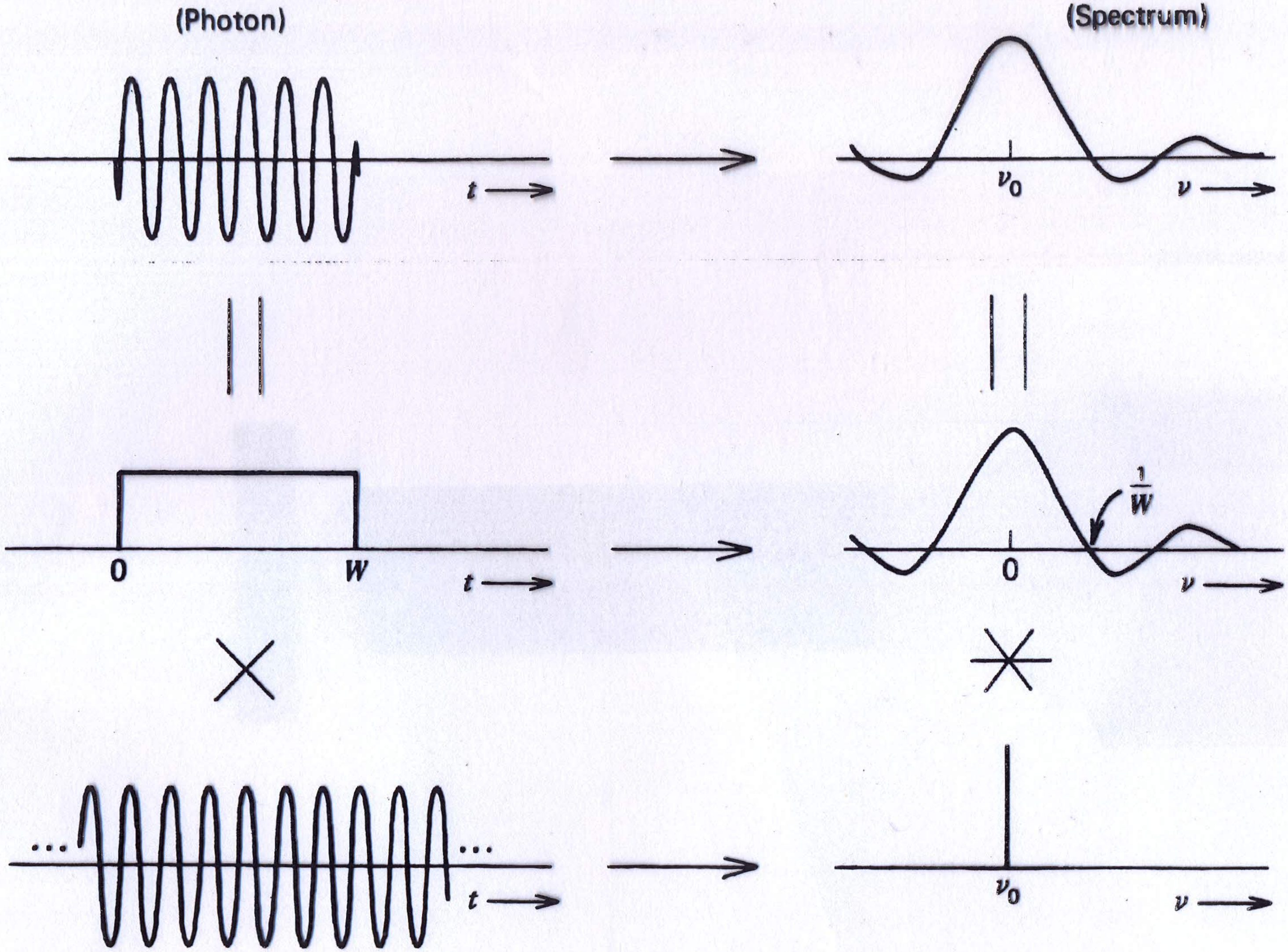


Fig. 11.3. Any real photon has a finite length of a few centimeters or $\approx 10^{-9}$ s, and a simplified way to view this is shown here. Specifically, the duration of the photon determines the width of the spectral line it produces.

b. Quasi-static. → *atom radiating in the average scalar field of perturbers*
perturbers are at random positions and slow.
Holtzmark approx.: level change by average external field

The first neighbor p = 2 model

probability of finding a charged neighbor
between *r* and *r* + *dr* :

$$W(r)\,dr \;=\; W(r)\frac{dr}{d\Delta\omega}\,d\Delta\omega$$

probability of finding a charged neighbor
between *r* and *r* + *dr* **and** not finding a neighbor
for $r' < r$

$$W(r)\,dr \;=\; \left(1-\int_0^r W(r')\,dr'\right) \;*\; a\,4\pi r^2 N\,dr$$

$$\frac{d}{dr}\left(\frac{W(r)}{4\pi a r^2 N}\right) = \frac{d}{dr}\left(1-\int_0^r W(r')\,dr'\right) = W(r)$$

$$\frac{d}{dr}\left(\frac{W(r)}{4\pi a r^2 N}\right) = -4\pi a r^2 N\left(\frac{W(r)}{4\pi a r^2 N}\right)$$

$$\frac{d}{dr}\left(\frac{W(r)}{4\pi a r^2 N}\right) = \frac{d}{dr}\left(1 - \int_0^r W(r')\,dr'\right)$$

$$\frac{d}{dr}\left(\frac{W(r)}{4\pi a r^2 N}\right) = -4\pi a r^2 N\left(\frac{W(r)}{4\pi a r^2 N}\right)$$

$$y(r) = \frac{W(r)}{4\pi a r^2 N} \qquad \frac{d}{dr}\left(y(r)\right) = -4\pi a r^2 N\left(y(r)\right)$$

$$y(r) = e^{-\frac{4}{3}\pi a r^3 N} + c; \qquad \lim_{r\to\infty} y(r) = 0; \qquad c = 0$$

there are no distant neighbors

$$W(r) = 4\pi r^2 N e^{-\frac{4}{3}\pi a r^3 N} \qquad (1)$$

probability density for nearest neighbor

$$\int_0^\infty W(r)\,dr = 1; \qquad a=1$$

there are at least one (and first) neighbor

$$\bar{r} = \int_0^\infty W(r)\,r\,dr \Big/ \int_0^\infty W(r)\,dr = 1$$

$$\bar{r} = \int_0^\infty 4\pi r^3 N e^{-\frac{4}{3}\pi r^3 N}\,dr = \frac{\Gamma(4/3)}{(4/3\pi N)^{1/3}} \simeq 0.55\,N^{-1/3}$$

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for comparison:

$$N = \left(\frac{4}{3}\pi r_0^3\right)^{-1}; \qquad r_0 = 0.62\,N^{-1/3}$$

radius → *energy* → *frequency*

$$\Delta\omega = \frac{C_{k,p}}{r^p} \quad \rightarrow \quad \frac{r_0}{r} = \left(\frac{\Delta\omega}{\Delta\omega_0}\right)^{\frac{1}{p}}$$

with $$r = \left(\frac{\Delta\omega}{\Delta\omega_0}\right)^{-1/p} r_0; \quad dr = -r_0\left(\frac{1}{p}\right)\left(\frac{\Delta\omega}{\Delta\omega_0}\right)^{-1/p-1} d\left(\frac{\Delta\omega}{\Delta\omega_0}\right)$$

rep N in eq. 1: $$W(r)\,dr = 4\pi r^2 \frac{3}{4\pi r_0^3} e^{-\frac{4}{3}\pi r^3 \frac{3}{4\pi r_0^3}} dr$$

radius → *energy* → *frequency*

$$\Delta\omega = \frac{C_{k,p}}{r^p} \quad \rightarrow \quad \frac{r_0}{r} = \left(\frac{\Delta\omega}{\Delta\omega_0}\right)^{\frac{1}{p}}$$

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rep N in eq. 1: $$W(r)\,dr = 4\pi r^2 \frac{3}{4\pi r_0^3} e^{-\frac{4}{3} r^3 \frac{3}{4\pi r_0^3}} dr$$

$$W(r)\,dr = 3\left(\frac{r}{r_0}\right)^3 e^{-\left(\frac{r}{r_0}\right)^3} \frac{dr}{r}$$

with frequency terms for *r* and *dr*:

$$W(\Delta\omega)\,d\Delta\omega \;=\; \frac{3}{p}\left(\frac{\Delta\omega_0}{\Delta\omega}\right)^{3/p+1} e^{-\left(\frac{\Delta\omega_0}{\Delta\omega}\right)^{3/p}} d\left(\frac{\Delta\omega}{\Delta\omega_0}\right)$$

for p = 2:

$$W(\Delta\omega)\,d\Delta\omega \;=\; \frac{3}{2}\left(\frac{\Delta\omega_0}{\Delta\omega}\right)^{\frac{5}{2}} e^{-\left(\frac{\Delta\omega_0}{\Delta\omega}\right)^{3/2}} d\left(\frac{\Delta\omega}{\Delta\omega_0}\right)$$

with $\beta=\dfrac{\Delta\omega}{\Delta\omega_0}$ and $W(\Delta\omega)\,d\Delta\omega \;\propto\; a_\nu(\Delta\omega)\,d\,\dfrac{\Delta\omega}{\Delta\omega_0}$

$$I(\beta)\,d\beta \;\propto\; \frac{3}{2}\beta^{-\frac{5}{2}} e^{-\beta^{-3/2}}\,d\beta$$

$\beta \rightarrow \infty$

$$I(\beta) \;\propto\; \frac{3}{2}\beta^{-\frac{5}{2}} \quad \neq \quad \beta^{-2}(impact\,;natural)$$

further neighbors where calculated up to Debye by Holtzmark

→ up to 50% difference in the far wings

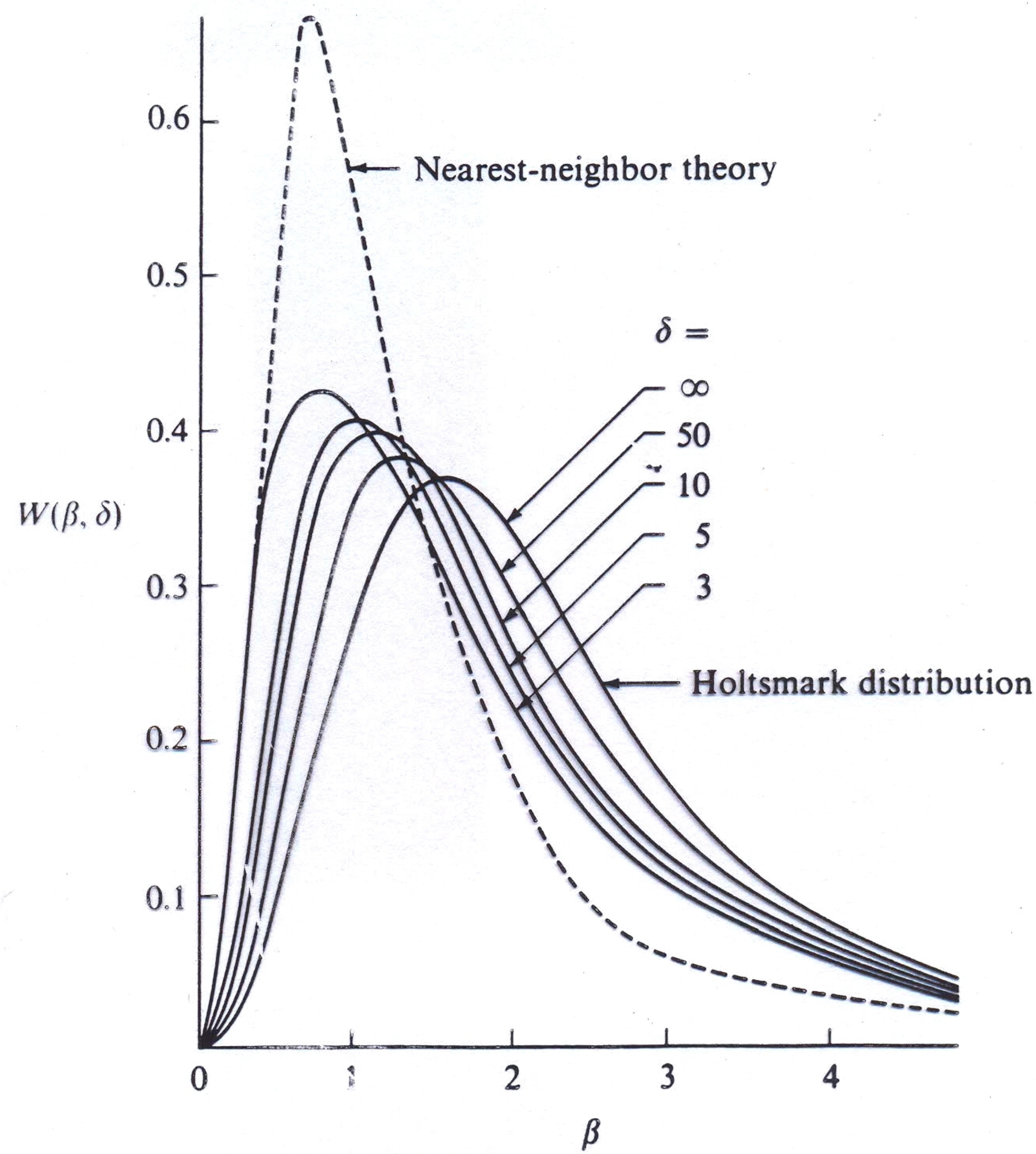
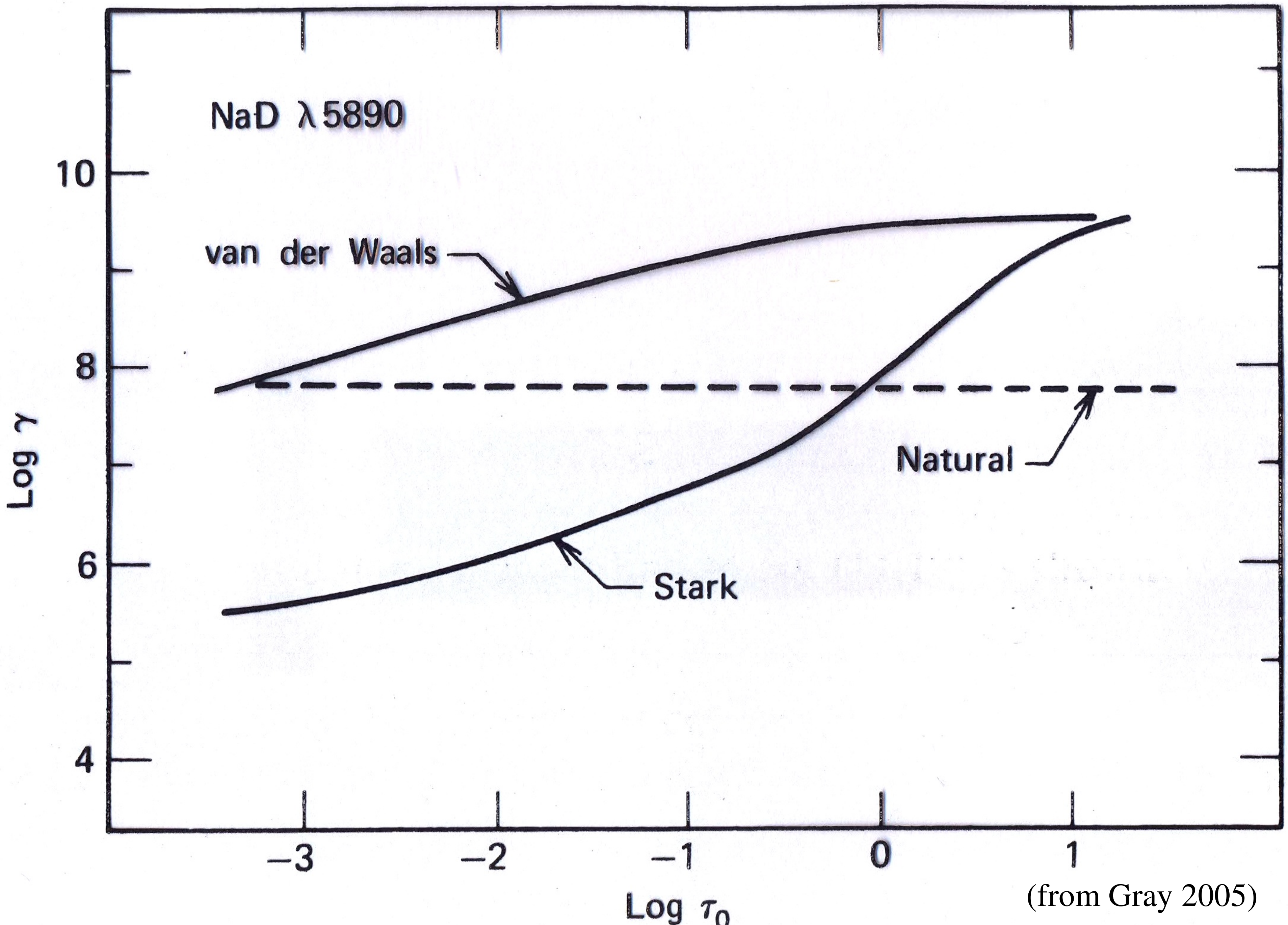


FIG. 9-2. Field distribution at a test point, including shielding effects; δ is the number of charged particles within the Debye sphere. (From G. Ecker, *Z. Ph.*, **148**, 593, 1957; by permission.)



(from Gray 2005)