

# Lecture 9

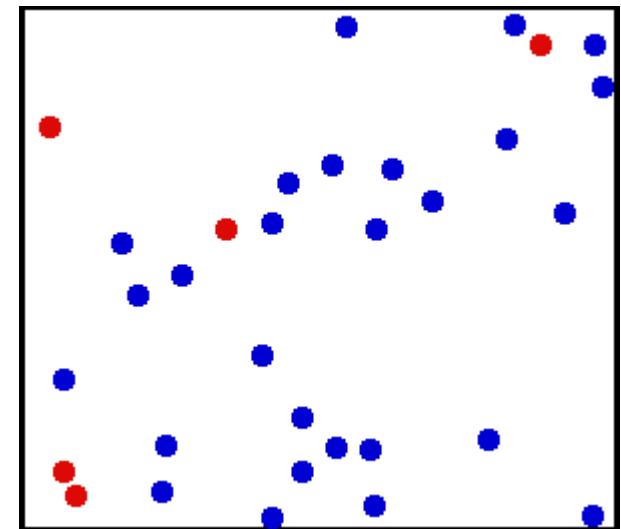
treasure map:

Gray: pg 236

H&M: pg 228

Böhm-Vitense: pg 117

Rutten: pg 116



# Microscopic Line Broadening

## I. Natural line profile

$$P_j(t) = \psi_j^* \psi_j e^{-A_{ji}t} \quad \text{with} \quad A_{ji} = \frac{1}{\Delta t_{ji}}$$

*j state probability and life time*

$$\psi_j(r,t) e^{-iw_{ji}t/\hbar} \rightarrow u_j e^{-iE_{ji}t/\hbar} e^{-A_{ji}t/2} \quad \text{with} \quad E_{ji} = \omega_0/\hbar$$

*(similar to classical damped osc.)*

$$|\mathcal{F}[\psi(t)]|^2 = I(\omega) = \frac{A_{ji}/2\pi}{(\omega - \omega_0)^2 + (A_{ji}/2)^2}$$

from  $A_{ji}$  to actual  $\Gamma_{ji}$ :

$$I(\omega) = \frac{\Gamma_{ji}/2\pi}{(\omega - \omega_0)^2 + (\Gamma_{ji}/2)^2}$$

sublevel probabilities summed (see H&M 2015)

$$\Delta\omega = \Gamma_{ji} / 2 \sim (10^{-8} \text{ s})^{-1}$$

absorption cross-section per atom:

$$a_v = \frac{\pi e^2}{mc} f_{ij} \frac{\Gamma_{ji}/4\pi^2}{(\nu - \nu_0)^2 + (\Gamma_{ji}/4\pi)^2}$$

## II. Thermal Doppler broadening

the Voigt profile

$$a_v(v) \rightarrow a_v\left(v - \frac{V_{part} v_0}{c}\right)$$

where  $V_{part}$  = particle thermal velocity

$v$  = obs. frequency

$v_0$  = line frequency

with

$$V_0^2 = V_x^2 + V_y^2 + V_z^2$$

$$\frac{\bar{m} v_0^2}{2} = \frac{3 k T}{2}$$

$$V_x^2 = \frac{k T}{\bar{m}} \rightarrow \frac{\Delta v_D}{v_0} = \frac{1}{c} \left( \frac{k T}{\bar{m}} \right)^{1/2}$$

$$W(T, V_x) dV = \left( \frac{2\pi kT}{\bar{m}} \right)^{-1/2} e^{-\frac{V_x^2 \bar{m}}{2kT}} dV \quad \text{Maxwell-Boltzmann probability}$$

$$a_v(v) = \left( \frac{m}{2\pi kT} \right)^{1/2} \left( \frac{\pi c^2}{mc} \right) f_{ij} \int_{-\infty}^{\infty} \frac{\left( \Gamma_{ji}/4\pi^2 \right) e^{-\frac{V_x^2}{2v_D^2}}}{\left( v - v_0 - \frac{V_x v_0}{c} \right)^2 + \left( \Gamma_{ji}/4\pi \right)^2} dV$$

$$a = \frac{\Gamma}{4\pi\Delta v_D}; \quad b = \frac{v - v_0}{\Delta v_D}; \quad y = \frac{V_x}{(kT/\bar{m})^{1/2}};$$

$$a_{\nu}(v) = \frac{\pi^{1/2} e^2}{mc} f_{ij} H \frac{(a, b)}{\Delta v_D}$$

with

$$H(a, b) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2}}{a^2 + (b - y)^2} dy$$

Voigt function

i.  $b \rightarrow \infty$  - far line wings

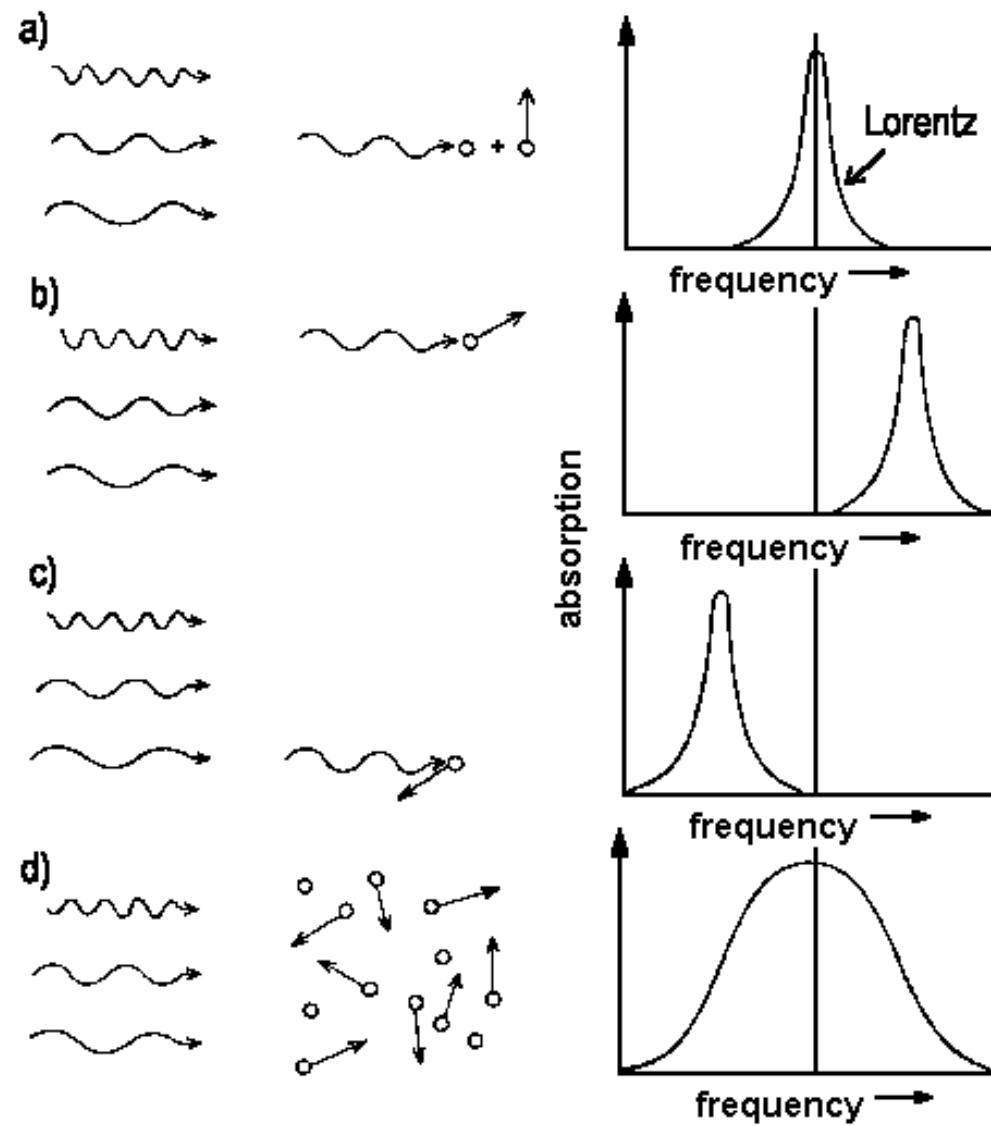
$$H(a, b) \rightarrow \sim \frac{a}{\pi^{1/2} b^2} \propto \frac{a}{(v - v_0)^2}$$

ii.  $\Delta v_D \gg \Gamma \rightarrow a \rightarrow 0$

all terms with  $b$  and  $y$  remain:

$$H(a, b) \rightarrow \sim e^{-b^2} \propto e^{-[(v - v_0)/\Delta v_D]^2}$$

gaussian core with thermal width



from Gericke 2015

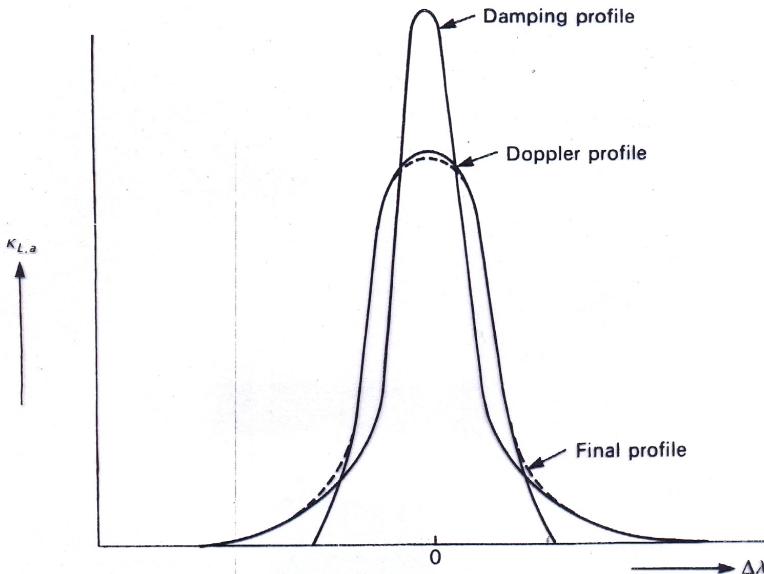


Fig. 10.6. A schematic plot of the different line profiles contributing to the Voigt profile. Generally, the line profile consists of the Doppler core and the damping wings. It is called a Voigt profile.

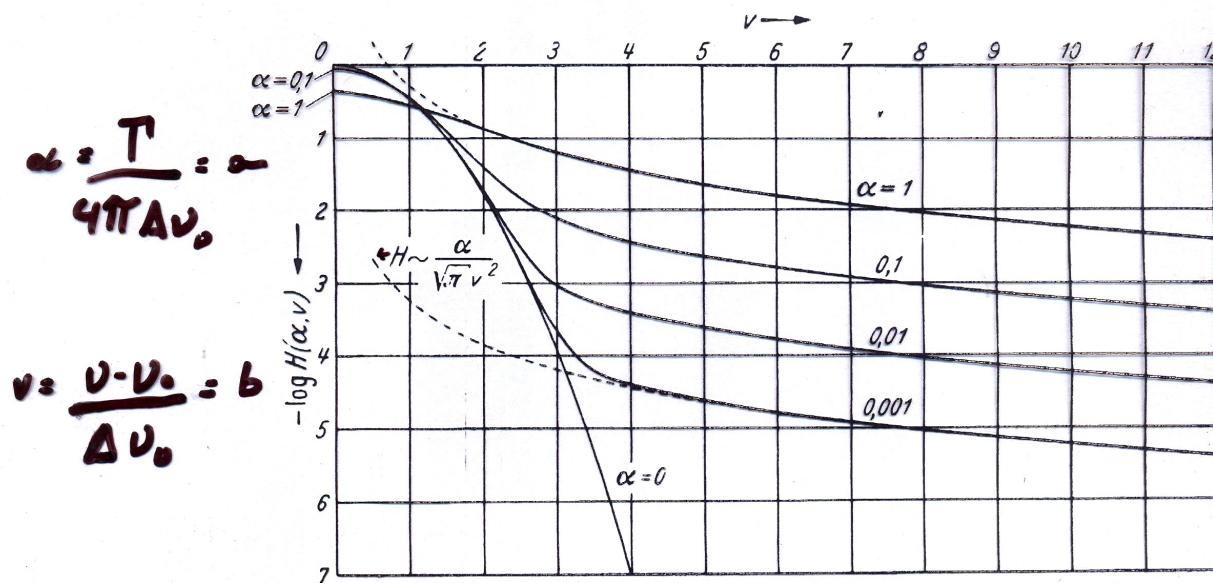


Fig. 10.7. Voigt profiles for different values of  $\alpha$  are shown in a logarithmic plot.  
 (From Unsöld, 1955, p. 265.)

from Bohn-Vitense 1989