Stellar atmospheres

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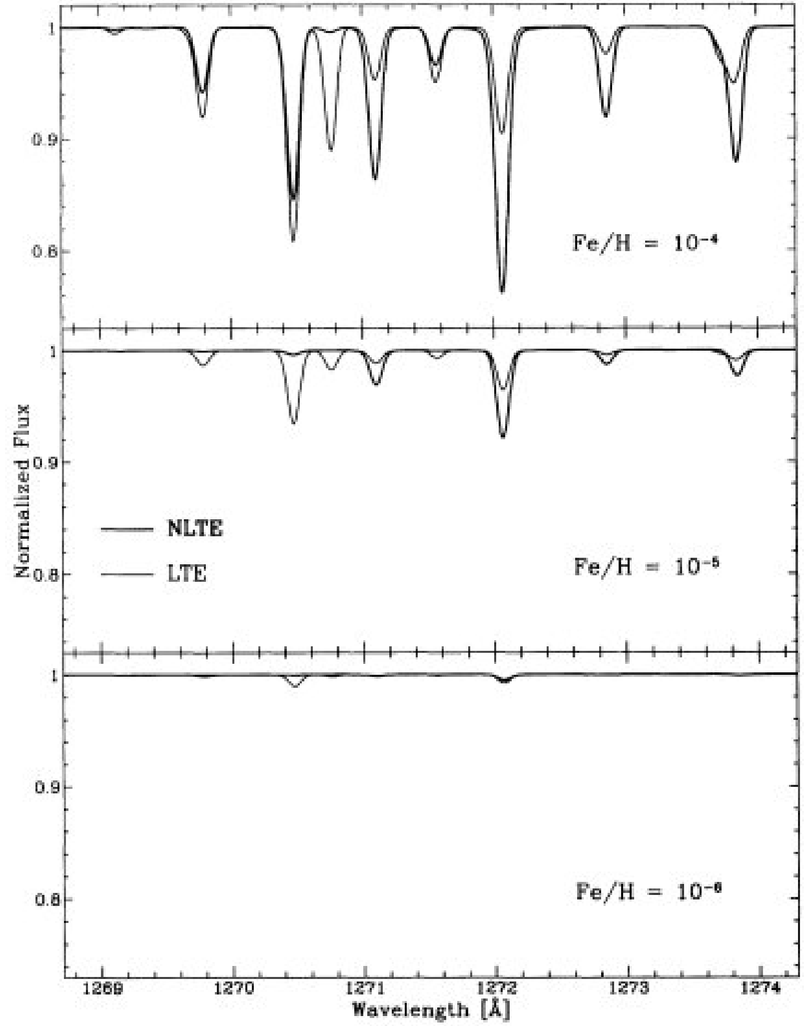
IAG-USP 2024



NLTE effects (even) in moderate temperature, high gravity atmospheres

from Lanz & Hubeny. 1995

Lecture 5

treasure map:

H&M: pg 262

Rutten: pg 89

Vitense: pg 163

Gray: pg 330

Oxenius: pg 1 *in "The Kinetic Theory of Particles and Photons", Springer-Verlag, 1986)*

The concept of Local Thermodynamic Equilibrium in stellar atmospheres

standard thermodynamics relations hold locally
for temperature and particle density

Equilibrium values apply locally to particles.

(while the radiation field **may** depart from a blackbody)

➡ *the particle ensemble properties depend only on T, n_e or N by:*

i. *Maxwellian velocity distribution, under energy equipartition in a single temperature system*

ii. *Boltzmann excitation equation*

iii. *Saha / Hoff ionization / dissociation equation*

$$f(\mathbf{v})\mathrm{d}\mathbf{v} = (m/2\pi kT)^{3/2}\ \exp(-mv^2/2kT)\,\mathrm{d}\mathbf{v} \tag{i}$$

$$(n_j/n_i) = (g_j/g_i)\ \exp\left[-(E_j - E_i)/kT\right] \tag{ii}$$

$$\frac{N_I}{N_{I+1}} = n_\mathrm{e}\,\frac{U_I}{U_{I+1}}\,C\,T^{-3/2}\ \exp(\chi_I/kT) \tag{iii}$$

with

$g_{i,j} = 2J_{i,j} + 1$

$$U = \sum_1^\infty g_i \exp(-E_i/kT)$$

$$C = (h^2/2\pi mk)^{3/2}$$

LTE from microscopic standpoint

LTE is fully attained with *Detailed Balance*

A → B ≡ B → A (radiative or collisional)

transition rates are equal in direct and inverse transitions.

Intensity asymmetry → radiative d.b. break (NLTE)

(particles may remain in d.b. while in NLTE – M. B. distr. holds)

conditions for NTLE: $A_{j,i} >> C_{j,i}$ @low n_e

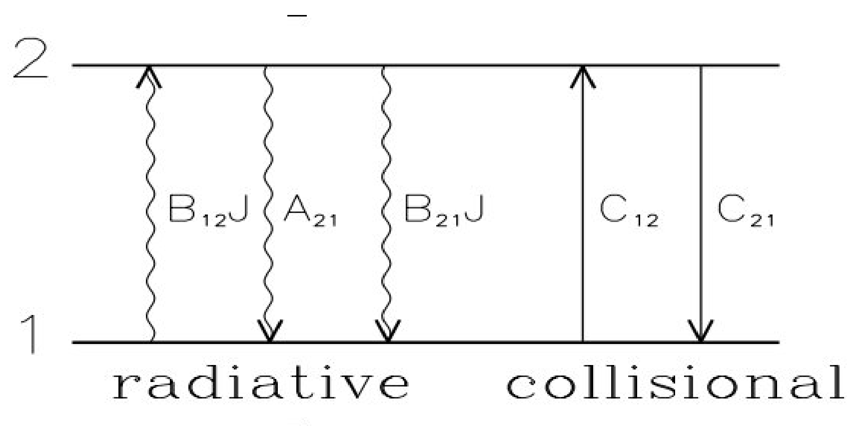
$f_\nu \neq B_\nu$

Two level atom example:

a) $C_{1,2} \propto n \langle v \rangle \propto n\, T^{1/2}$

b) $J \propto T^b$ e.g. b = 4 (LTE)

c) $l_\nu \propto (\sigma_\nu n)^{-1}$



$$\frac{\mathrm{d}n_i(\vec{r})}{\mathrm{d}t} = \sum_{j\neq i}^{N} n_j(\vec{r}) P_{ji}(\vec{r}) - n_i(\vec{r}) \sum_{j\neq i}^{N} P_{ij}(\vec{r}) = 0$$

(Statistical equilibrium or rate equation)

with

$$P_{i,j} = C_{i,j} + B_{i,j}\, J + A_{i,j}$$

$$P_{j,i} = C_{j,i} + B_{j,i}\, J + A_{j,i}$$

$$\beta_i \equiv \frac{n_i}{n_{i\ LTE}}$$

Zwaan departure coef.

Model atmospheres

LTE atmosphere with true absorption + scattering

$k_\nu = k_{abs} + \sigma_{sct}$

and thermal emission + for coherent and isotropic scattering:

$\eta_\nu = k_{abs} B_\nu + \sigma_{sct} J_\nu$

➡ $S_\nu = (k_{abs} B_\nu + \sigma_{sct} J_\nu) \ / \ (k_{abs} + \sigma_{sct})$

with $\epsilon_\nu = \frac{k_{absorption}}{(k_{absorption} + \sigma_{sct})}$ thermal coupling parameter

$S_\nu = \varepsilon_\nu B_\nu + (1 - \varepsilon_\nu) J_\nu$ ⟶ Milne-Eddington r.t. eq.

The Variable Eddington Factor (VEF) method

by

Auer & Mihalas (1970)

$$S(\tau) = B(\tau) \; (k=0)$$

$$S_\nu \, (\tau_\nu)$$

$$S_\nu = \varepsilon_\nu B_\nu + (1 - \varepsilon_\nu) J_\nu$$

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$$

$$I_\nu(\tau_\nu, \mu)$$

$$J_\nu = \frac{1}{2}\int_{-1}^{1} I_\nu(\mu)\mathrm{d}\mu \; ,$$

$$H_\nu = \frac{1}{2}\int_{-1}^{1} \mu \, I_\nu(\mu)\mathrm{d}\mu$$

$$K_\nu = \frac{1}{2}\int_{-1}^{1} \mu^2 I_\nu(\mu)\mathrm{d}\mu$$

$$f_\nu^{k-1} = \left(K_\nu / J_\nu\right)_{k-1}$$

$$\frac{\mathrm{d}H_\nu}{\mathrm{d}\tau_\nu} = J_\nu - S_\nu$$

$$\frac{\mathrm{d}K_\nu}{\mathrm{d}\tau_\nu} = H_\nu \; .$$

$$J_\nu \, (\tau_\nu)$$

$$f_\nu^K = 1/3; \quad k=0$$

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$f_\nu^K = 1/3; \quad k=0$

Model atmospheres

The lambda operator

Complete formal solution in 2 currents with any *S(τ)*:

$$I_{+(\tau,\mu)} = \frac{1}{\mu}\int_{\tau}^{\infty} S(\tau')\,e^{-\frac{(\tau'-\tau)}{\mu}}\,d\tau' \qquad \mu \geq 0$$

$$I_{-(\tau,\mu)} = I(0)\,e^{\frac{\tau}{\mu}} + \frac{1}{\mu}\int_{\tau}^{0} S(\tau')\,e^{-\frac{(\tau'-\tau)}{\mu}}\,d\tau' \qquad \mu < 0$$

$$J(\tau) = \frac{1}{2}\left[\int_{-1}^{0} I_{-(\tau,\mu)}\,d\mu + \int_{0}^{1} I_{+(\tau,\mu)}\,d\mu\right]$$

$$= \frac{1}{2}\int_{0}^{1}\int_{\tau}^{\infty}\frac{1}{\mu}S(\tau')e^{-\frac{(\tau'-\tau)}{\mu}}d\tau' d\mu \; - \; \frac{1}{2}\int_{-1}^{0}\int_{0}^{\tau}\frac{1}{\mu}S(\tau')e^{-\frac{(\tau'-\tau)}{\mu}}d\tau' d\mu$$

$$= \frac{1}{2}\int_0^1 \int_\tau^\infty \frac{1}{\mu} S(\tau') e^{-\frac{(\tau'-\tau)}{\mu}} d\tau' d\mu - \frac{1}{2}\int_{-1}^0 \int_0^\tau \frac{1}{\mu} S(\tau') e^{-\frac{(\tau'-\tau)}{\mu}} d\tau' d\mu$$

$$\omega = -\frac{1}{\mu}; \quad \frac{d\mu}{\mu} = -\frac{d\omega}{\omega}$$

with

$$\omega' = \frac{1}{\mu}; \quad \frac{d\mu}{\mu} = -\frac{d\omega'}{\omega'}$$

$$= -\frac{1}{2}\int_\infty^1 \int_\tau^\infty \frac{1}{\omega'} S(\tau') e^{-\omega'(\tau'-\tau)} d\tau' d\omega' + \frac{1}{2}\int_1^\infty \int_0^\tau \frac{1}{\omega} S(\tau') e^{-\omega(\tau'-\tau)} d\tau' d\omega$$

$$= \frac{1}{2}\int_0^1\int_\tau^\infty \frac{1}{\mu}S(\tau')e^{-\frac{(\tau'-\tau)}{\mu}}d\tau' d\mu - \frac{1}{2}\int_{-1}^0\int_0^\tau \frac{1}{\mu}S(\tau')e^{-\frac{(\tau'-\tau)}{\mu}}d\tau' d\mu$$

with

$$\omega=-\frac{1}{\mu};\qquad \frac{d\mu}{\mu}=-\frac{d\omega}{\omega}$$

$$\omega'=\frac{1}{\mu};\qquad \frac{d\mu}{\mu}=-\frac{d\omega'}{\omega'}$$

and

$$E_n(x)=\int_1^\infty \frac{e^{-xt}}{t^n}dt$$

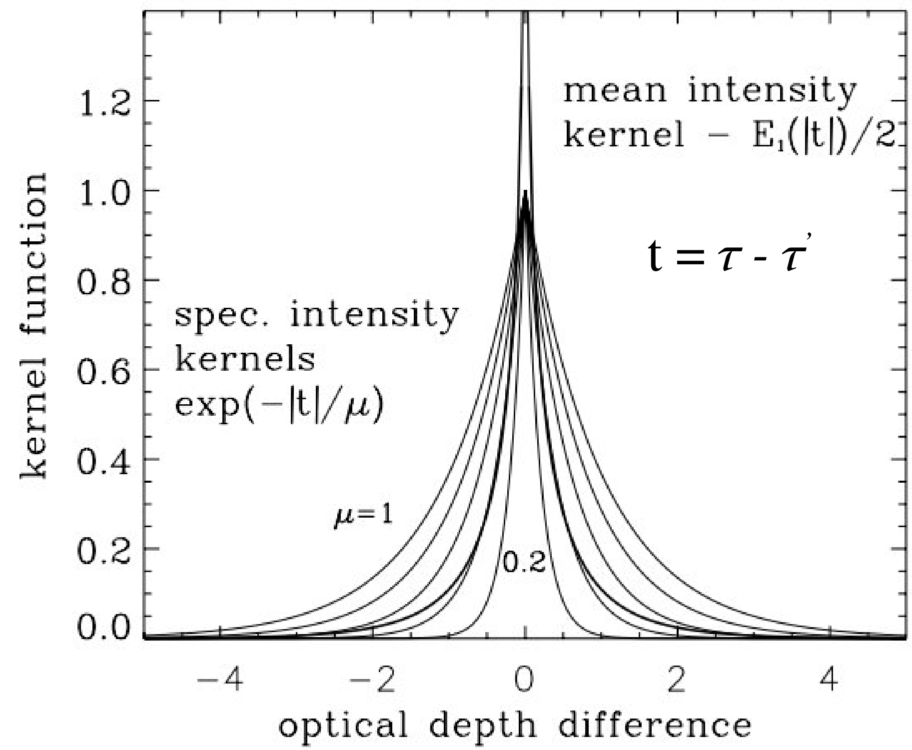
$$= -\frac{1}{2}\int_\infty^1\int_\tau^\infty \frac{1}{\omega'}S(\tau')e^{-\omega'(\tau'-\tau)}d\tau' d\omega' + \frac{1}{2}\int_1^\infty\int_0^\tau \frac{1}{\omega}S(\tau')e^{-\omega(\tau'-\tau)}d\tau' d\omega$$

$$J(\tau) = \frac{1}{2}\left(\int_0^\tau S(\tau')E_1(\tau-\tau')d\tau' + \int_\tau^\infty S(\tau')E_1(\tau'-\tau)d\tau'\right)$$

$$J(\tau) = \frac{1}{2}\int_0^\infty S(\tau')E_1(|\tau-\tau'|)d\tau'$$

$$J(\tau_\nu) = \Lambda_{\tau_\nu}\left[S(\tau_\nu')\right]$$

$$I_{+(\tau_\nu,\mu)} = \frac{1}{\mu}\int_{\tau_\nu}^\infty S(\tau')e^{-\frac{(\tau'-\tau_\nu)}{\mu}}d\tau'$$



from Hubeny 1993

$$J(\tau) = \frac{1}{2}\int_0^{\infty} S(\tau')E_1(|\tau-\tau'|)\,d\tau'$$

$$J(\tau_\nu) = \int_0^{\infty} \Lambda_{\tau_\nu}\left[S(\tau_\nu')\right]d\tau_\nu'$$

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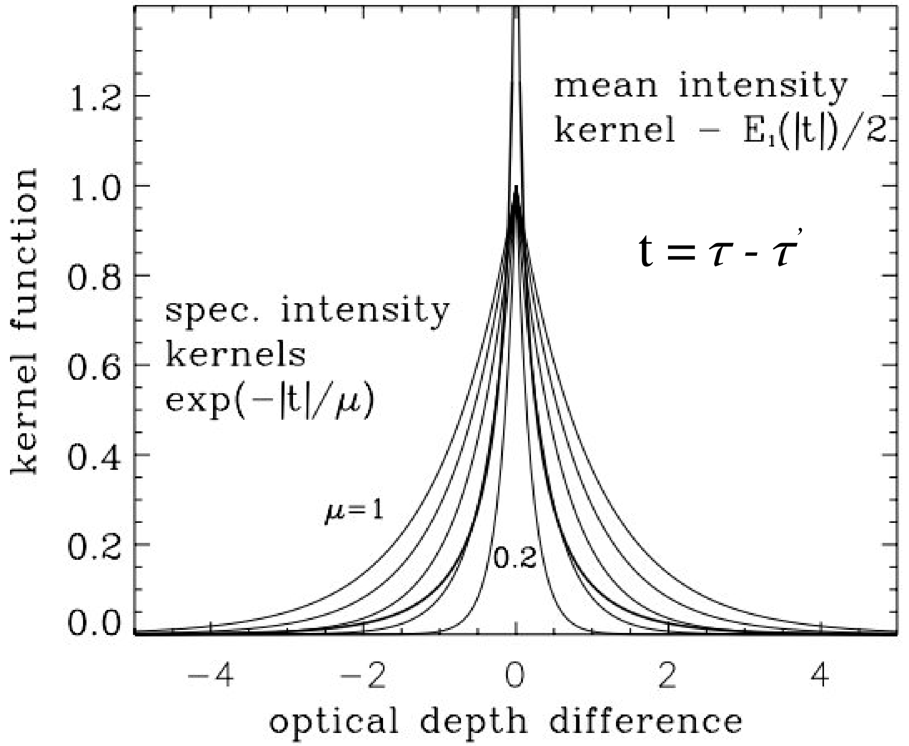

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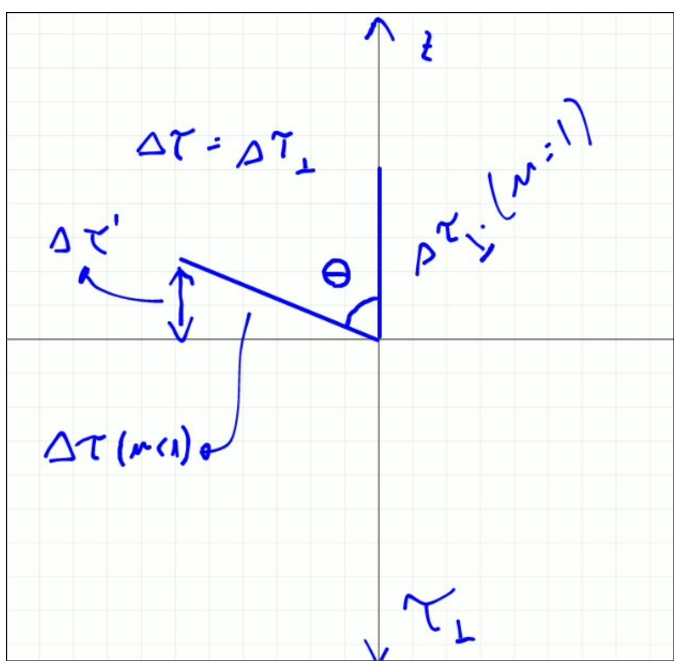

discrete optical depth sum: $J_d = \sum_{d'=1}^{D} \Lambda_{dd'} S_{d'}$

with $S(\tau) = \delta(\tau - \tau_i)$:

$$\begin{pmatrix} J_1 \\ J_2 \\ \vdots \\ J_D \end{pmatrix} = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} & \dots & \Lambda_{1D} \\ \Lambda_{21} & \Lambda_{22} & \dots & \Lambda_{2D} \\ \vdots & \vdots & \ddots & \vdots \\ \Lambda_{D1} & \Lambda_{D2} & \dots & \Lambda_{DD} \end{pmatrix} \times \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \end{pmatrix} = \begin{pmatrix} \Lambda_{1i} \\ \Lambda_{2i} \\ \vdots \\ \Lambda_{Di} \end{pmatrix}$$