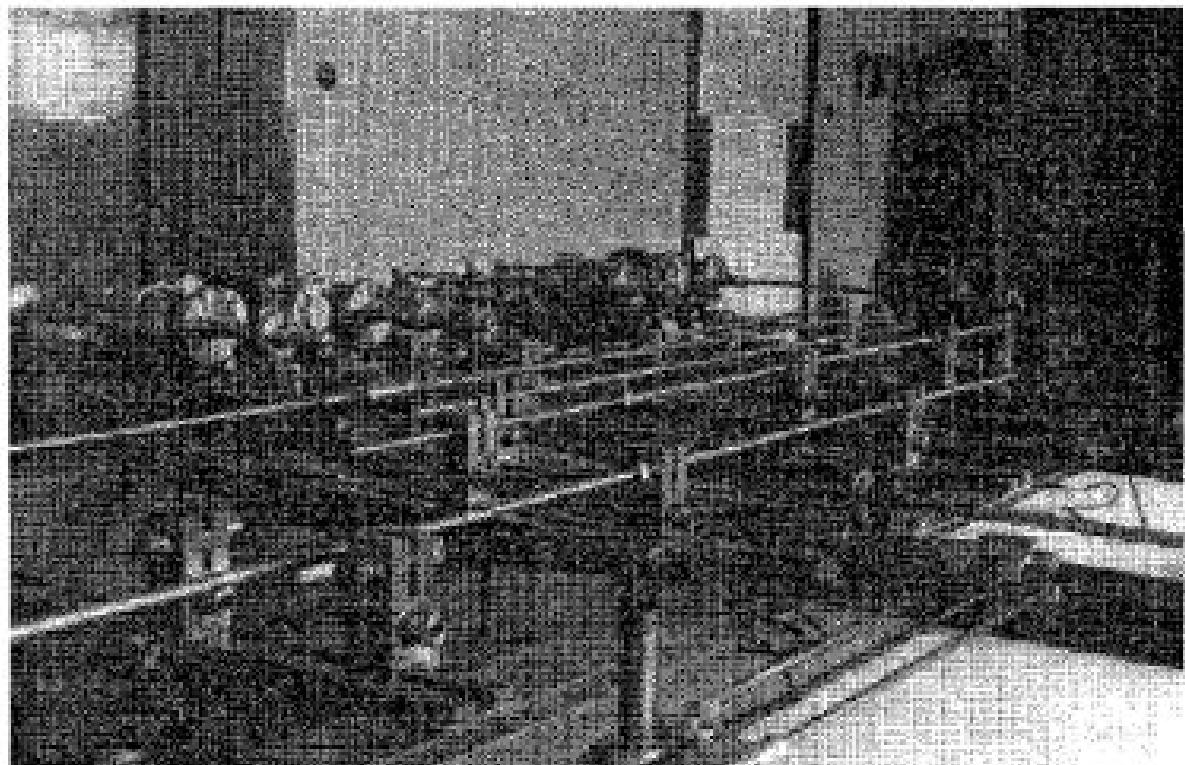


Atmosferas Estelares

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IAG-USP 2024



The Oslo analyzer by S. Rosseland (1938)

<https://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=539912>

Lecture 4

treasure map:

H&M: pg 374, 347

Rutten: pg 30

Vitense: pg 46

Gray: pg 139

The Diffusion Approximation

$$S_\nu \rightarrow B_\nu ; \quad \tau_\nu \rightarrow \infty$$

expansion around an arbitrary (as good as large) τ_ν :

$$S_\nu(t_\nu) = \sum_{n=0}^{\infty} \frac{d^n B_\nu}{d\tau_\nu^n} \frac{(t_\nu - \tau_\nu)^n}{n!} \quad \text{into the formal solution in 2 currents:}$$

$$I_\nu(\tau_\nu, \mu) = \int_{\tau_\nu}^{\infty} S_\nu(t) e^{-(t-\tau_\nu)/\mu} dt/\mu, \quad \text{for } \mu \geq 0$$

$$I_\nu(\tau_\nu, \mu) = \int_0^{\tau_\nu} S_\nu(t) e^{-(\tau_\nu-t)/(-\mu)} dt/(-\mu), \quad \text{for } \mu < 0$$

The Diffusion Approximation (cont.)

taking the outward ($0 < \mu < 1$) intensity at τ_v :

$$I_v(\tau, \mu) = \int_0^\infty \sum \frac{(t - \tau_v)^n}{n!} \frac{d^n B_v}{d\tau^n} e^{\frac{-(t - \tau_v)}{\mu}} \frac{dt}{\mu} \quad \text{with } x = (t - \tau_v)$$

The Diffusion Approximation (cont.)

taking the outward ($0 < \mu < 1$) intensity at τ_v :

$$I_v(\tau, \mu) = \int_0^\infty \sum \frac{(t - \tau_v)^n}{n!} \frac{d^n B_v}{d\tau^n} e^{\frac{-(t-\tau_v)}{\mu}} \frac{dt}{\mu} \quad \text{with } x = (t - \tau_v)$$

$$I_v(\tau, \mu) = \sum \frac{d^n B_v}{d\tau^n} \int_0^\infty \frac{x^n}{n!} e^{-\frac{x}{\mu}} \frac{dx}{\mu}$$

The Diffusion Approximation (cont.)

taking the outward ($0 < \mu < 1$) intensity at τ_v :

$$I_v(\tau, \mu) = \int_{\tau_v}^{\infty} \sum \frac{(t - \tau_v)^n}{n!} \frac{d^n B_v}{d\tau^n} e^{\frac{-(t - \tau_v)}{\mu}} \frac{dt}{\mu} \quad \text{with } x = (t - \tau_v)$$

$$I_v(\tau, \mu) = \sum \frac{d^n B_v}{d\tau^n} \int_0^{\infty} \frac{x^n}{n!} e^{-\frac{x}{\mu}} \frac{dx}{\mu}$$

$$I_v(\tau, \mu) = \sum \frac{d^n B_v}{d\tau^n} \mu^n = B_v(\tau) + \mu \frac{dB_{nu}}{d\tau}. \dots$$

The Diffusion Approximation (cont.)

error in diffusive approximation

convergence of the expansion

$$\frac{\delta S_{v,n}}{S_v} \propto \frac{\frac{d^{n+2} B_v}{d \tau^{n+2}}}{\frac{d^n B_v}{d \tau^n}} \sim \frac{\frac{B_v^{n+2}}{\tau^{n+2}}}{\frac{B_v^n}{\tau^n}} \sim \tau^{-2}$$

The Diffusion Approximation (cont.)

The diffusion approx. in the higher radiation moments

- multiply by μ^j and integrate over μ .

$$J_\nu(\tau, \mu) = \int_{-1}^1 \left(B_\nu(\tau) + \mu \frac{dB_\nu}{d\tau} + \mu^2 \frac{d^2 B_\nu}{d\tau^2} + \dots \right) d\mu$$

all odd terms on μ vanish

$$J_\nu(\tau_\nu) = B_\nu(\tau_\nu) + \frac{1}{3} \frac{d^2 B_\nu}{d\tau_\nu^2} + \dots \quad j = 0$$

$$H_\nu(\tau_\nu) = \frac{1}{3} \frac{dB_\nu}{d\tau_\nu} + \dots, \quad j = 1$$

$$K_\nu(\tau_\nu) = \frac{1}{3} B_\nu(\tau_\nu) + \frac{1}{5} \frac{d^2 B_\nu}{d\tau_\nu^2} + \dots \quad j = 2$$

The Diffusion Approximation (cont.)

The diffusion approx. in the higher radiation moments

deep enough:

as $S_\nu \rightarrow B_\nu$:

$J_\nu \rightarrow B_\nu$

$J_\nu / K_\nu \rightarrow 3$

$$J_\nu(\tau_\nu) = B_\nu(\tau_\nu) + \frac{1}{3} \frac{dB_\nu}{d\tau_\nu^2} + \dots$$

$$H_\nu(\tau_\nu) = \frac{1}{3} \frac{dB_\nu}{d\tau_\nu} + \dots,$$

$$K_\nu(\tau_\nu) = \frac{1}{3} B_\nu(\tau_\nu) + \frac{1}{5} \frac{dB_\nu}{d\tau_\nu^2} + \dots$$

$$H_\nu = \frac{1}{3} \frac{dB_\nu}{d\tau_\nu} = -\frac{1}{3} \frac{1}{\chi_\nu} \frac{dB_\nu}{dz} = -\frac{1}{3} \frac{1}{\chi_\nu} \frac{dB_\nu}{dT} \frac{dT}{dz}$$

where χ_ν is the linear opacity coef.

The Diffusion Approximation (cont.)

The diffusion coefficient for photons is:

$$-\frac{1}{3} \frac{1}{\chi_\nu} \frac{dB_\nu}{dT}$$

The bolometric flux is defined in terms of an appropriate opacity coefficient:

$$H = - \left(\frac{1}{3} \frac{1}{\chi_R} \frac{dB}{dT} \right) \frac{dT}{dz}$$

where χ_R is the Rosseland opacity coefficient

$$\frac{1}{\chi_R} \frac{dB}{dT} = \int_0^\infty \frac{1}{\chi_\nu} \frac{dB_\nu}{dT} d\nu$$

i. weight proportional to the flux
ii. exact flux (and temp.) structure @ diffusion
however...

iii. does not care about your favorite “ ν ”

Moments of the Transfer Equation

$$\left(\frac{1}{c} \frac{\partial}{\partial t} + \mathbf{n} \cdot \nabla \right) I(\mathbf{r}, \mathbf{n}, v, t) = \eta(\mathbf{r}, \mathbf{n}, v, t) - x(\mathbf{r}, \mathbf{n}, v, t) I(\mathbf{r}, \mathbf{n}, v, t)$$

μ^j integrated over solid angle:

$$\frac{\partial E_\nu}{\partial t} + \nabla \cdot \mathbf{F}_\nu = \eta_\nu - \chi_\nu c E_\nu \quad j=0$$

$$\frac{1}{c} \frac{\partial \mathbf{F}_\nu}{\partial t} + c \nabla \cdot \mathbf{P}_\nu = -\chi_\nu \mathbf{F}_\nu \quad j=1$$

With a symmetric emission coef. η

With astrophysical notation, changing to perpendicular optical depth:

$$\frac{dH_\nu}{d\tau_\nu} = J_\nu - S_\nu \quad (1)$$

open equation set

$$\frac{dK_\nu}{d\tau_\nu} = H_\nu . \quad (2)$$

with: $f_\nu^K \equiv K_\nu / J_\nu$ Eddington factor – predictor in iterative solutions

→ $\frac{d^2(f_\nu^K J_\nu)}{d\tau_\nu^2} = J_\nu - S_\nu$ angle-averaged quantities - only

The Grey Atmosphere

- i. The grey linear opacity coef. is the Rosseland harmonic mean.
- ii. The diffusive approximation for J extends over the whole atmosphere (the *Eddington approximation*).
- iii. The atmosphere is in strict radiative equilibrium.

from eq. 2 with Eddington factor = $\frac{1}{3}$ and considering bolometric quantities:

$$H(0) = \frac{1}{3} \frac{dJ}{d\tau}(\tau=0); \quad \text{with } \tau = -\chi_{Rosseland} dz$$

$$\frac{dH}{d\tau} = 0 = \frac{1}{3} \frac{d^2 J}{d\tau^2} \quad (\text{rad. equilibrium})$$

$$J = 3H(0)\tau + q; \quad \text{with } q = \text{Hopf function} \quad (1)$$

The Grey Atmosphere (cont.)

The observed $H(0)$ boundary condition fix the Hopf function

$$H(0) = \frac{1}{2} \int_{-1}^1 \mu I(0, \mu) d\mu = \frac{1}{2} \bar{\mu} \int_0^1 I(0, \mu) d\mu = \frac{1}{2} J(0)$$

(with $\bar{\mu} = 1/2$; $I(0, \mu) = 0$ for $\mu < 0$)

$$J(0) = q = 2H(0) ;$$

in 1: $J(\tau) = 3H(0)[\tau+2/3]$ (2) the grey τ structure

The Grey Atmosphere (cont.)

i. the temperature structure

$$H(0) = F(0)/4\pi = \frac{\sigma}{4\pi} T_{eff}^4$$

$$J(\tau) = \frac{3\sigma}{4\pi} T_{eff}^4 \left[\tau + \frac{2}{3} \right] ; \quad \text{with}$$

$$J(\tau) \cong \frac{\sigma}{\pi} T^4(\tau) \quad (\text{Eddington approx. } J \rightarrow B \text{ for any } \tau)$$

$$T^4(\tau) = \frac{3}{4} T_{eff}^4 \left[\tau + \frac{2}{3} \right]$$

The Grey Atmosphere (cont.)

ii. the angle dependence – a limb darkened atmosphere

the formal solution for semi-infinite atmosphere
with $S(\tau) = J(\tau)$

$$I(0, \mu) = \frac{1}{\mu} \int_0^{\infty} S(\tau') e^{\frac{-\tau'}{\mu}} d\tau'$$

$$I(0, \mu) = \frac{1}{\mu} \int_0^{\infty} 3H(0)[\tau' + 2/3] e^{\frac{-\tau'}{\mu}} d\tau'$$

The Grey Atmosphere (cont.)

ii. the angle dependence – a limb darkened atmosphere

the formal solution for semi-infinite atmosphere
with $S(\tau) = J(\tau)$

$$I(0, \mu) = \frac{1}{\mu} \int_0^{\infty} S(\tau') e^{\frac{-\tau'}{\mu}} d\tau'$$

$$I(0, \mu) = \frac{1}{\mu} \int_0^{\infty} 3H(0)[\tau' + 2/3] e^{\frac{-\tau'}{\mu}} d\tau'$$

$$I(0, \mu) = \frac{3H(0)}{\mu} \left[\int_0^{\infty} \tau' e^{\frac{-\tau'}{\mu}} d\tau' + \frac{2}{3} \int_0^{\infty} e^{\frac{-\tau'}{\mu}} d\tau' \right]$$

The Grey Atmosphere (cont.)

ii. the angle dependence – a limb darkened atmosphere

the formal solution for semi-infinite atmosphere
with $S(\tau) = J(\tau)$

$$I(0, \mu) = \frac{1}{\mu} \int_0^{\infty} S(\tau') e^{\frac{-\tau'}{\mu}} d\tau'$$

$$I(0, \mu) = \frac{1}{\mu} \int_0^{\infty} 3H(0)[\tau' + 2/3] e^{\frac{-\tau'}{\mu}} d\tau'$$

$$I(0, \mu) = \frac{3H(0)}{\mu} \left[\int_0^{\infty} \tau' e^{\frac{-\tau'}{\mu}} d\tau' + \frac{2}{3} \int_0^{\infty} e^{\frac{-\tau'}{\mu}} d\tau' \right]$$

$$I(0, \mu) = 3H(0) \left[\mu + \frac{2}{3} \right] = \frac{3\sigma T_{eff}^4}{4\pi} \left[\mu + \frac{2}{3} \right]$$

The Grey Atmosphere (cont.)

ii. the angle dependence – a linear limb darkened atmosphere

$$I(0, \mu) = 3H(0) \left[\mu + \frac{2}{3} \right]; \quad \text{with (2)} \quad J(\tau) = 3H(0) \left[\tau + \frac{2}{3} \right]$$

$$J(\tau) = I(0, \mu=\tau)$$

Eddington-Barbier relation with $S(\tau) = J(\tau)$

(linear LD and E-B relation found for non-arbitrary linear $S(\tau)$)