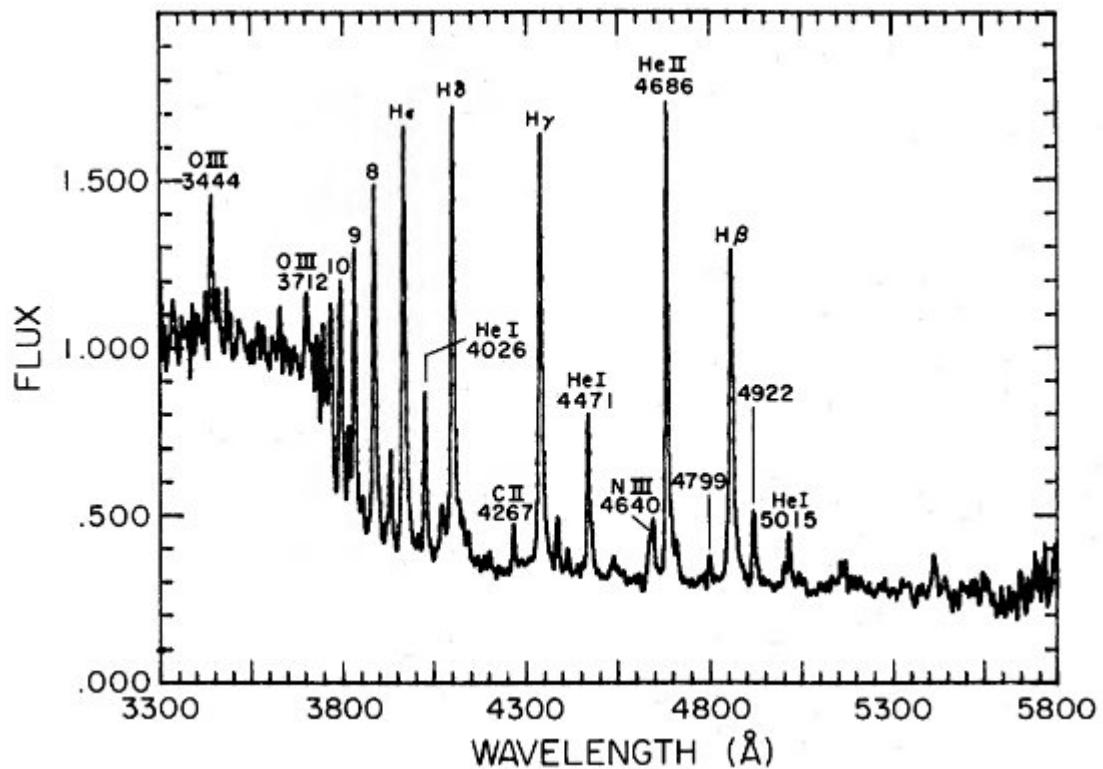


Atmosferas Estelares

prof. Marcos Diaz

IAG-USP 2024



Optically thick Balmer lines in the polar MR Ser

from Liebert et al. 1982

Lecture 3

treasure map:

H&M: pg 98

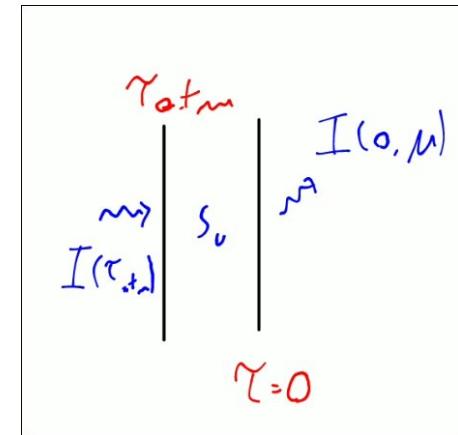
Rutten: pg 4

Vitense: pg 29

Finite slab atmosphere with $S_v = \text{constant}$
 (e.g. isothermal atmosphere)

from formal solution with: $\tau_2 = \tau_{\text{atm}}$, $\tau_1 = 0$:

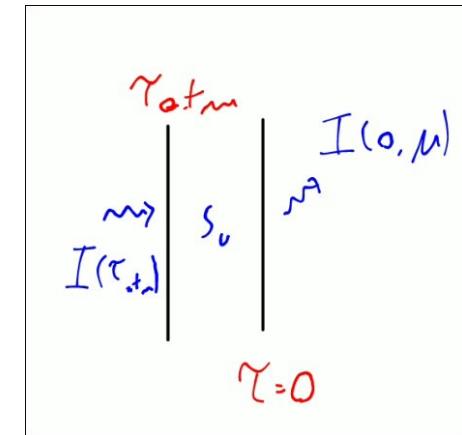
$$I_v(0) = I_v(\tau_{\text{atm}}) e^{\frac{-\tau_{\text{atm}}}{\mu}} + \frac{S_v}{\mu} \int_0^{\tau_{\text{atm}}} e^{\frac{-\tau'}{\mu}} d\tau'$$



i. Finite slab atmosphere with $S_v = \text{constant}$
 (e.g. isothermal atmosphere)

from formal solution with: $\tau_2 = \tau_{\text{atm}}$, $\tau_1 = 0$:

$$I_v(0) = I_v(\tau_{\text{atm}}) e^{\frac{-\tau_{\text{atm}}}{\mu}} + \frac{S_v}{\mu} \int_0^{\tau_{\text{atm}}} e^{\frac{-\tau'}{\mu}} d\tau'$$

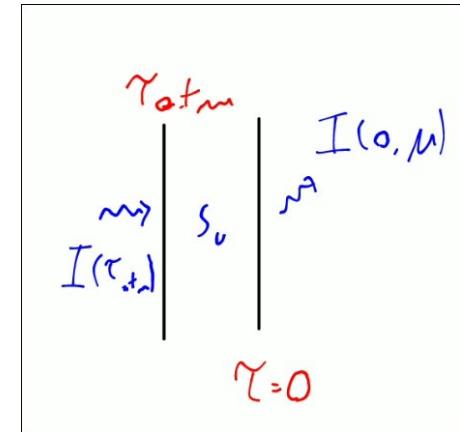


$$I_v(0) = I_v(\tau_{\text{atm}}) e^{\frac{-\tau_{\text{atm}}}{\mu}} - S_v e^{\frac{-\tau'}{\mu}}$$

Finite slab atmosphere with $S_v = \text{constant}$ (e.g. isothermal atmosphere)

from formal solution with: $\tau_2 = \tau_{\text{atm}}$, $\tau_1 = 0$:

$$I_v(0) = I_v(\tau_{\text{atm}}) e^{-\frac{\tau_{\text{atm}}}{\mu}} + \frac{S_v}{\mu} \int_0^{\tau_{\text{atm}}} e^{-\frac{\tau'}{\mu}} d\tau'$$



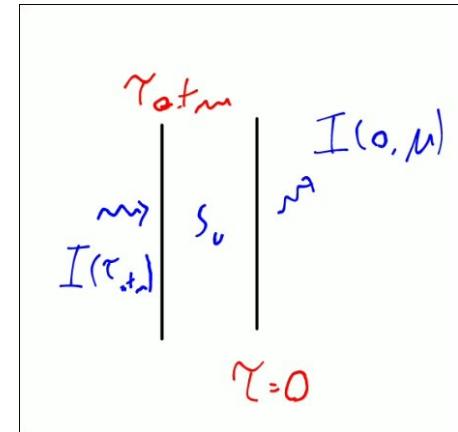
$$I_v(0) = I_v(\tau_{\text{atm}}) e^{-\frac{\tau_{\text{atm}}}{\mu}} - S_v e^{-\frac{\tau_{\text{atm}}}{\mu}}$$

$$I_v(0) = I_v(\tau_{\text{atm}}) e^{-\frac{\tau_{\text{atm}}}{\mu}} + S_v \left(1 - e^{-\frac{\tau_{\text{atm}}}{\mu}} \right)$$

Finite slab atmosphere with $S_v = \text{constant}$ (e.g. isothermal atmosphere)

from formal solution with: $\tau_2 = \tau_{\text{atm}}$, $\tau_1 = 0$:

$$I_v(0) = I_v(\tau_{\text{atm}}) e^{-\frac{\tau_{\text{atm}}}{\mu}} + S_v \left(1 - e^{-\frac{\tau_{\text{atm}}}{\mu}} \right)$$



i. $\tau_{\text{atm}} \gg 1 \rightarrow I_v(0) = S_v$

ii. $\tau_{\text{atm}} \ll 1 \rightarrow I_v(0, \mu=1) = [S_v - I_v(\tau_{\text{atm}})]\tau_{\text{atm}} + I_v(\tau_{\text{atm}})$

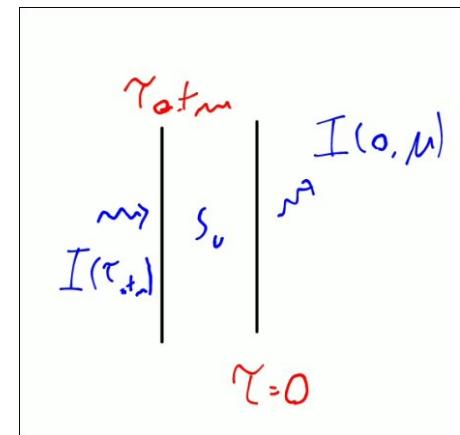
Kirchhoff's Laws

$\tau_{\text{atm}} < 1; \tau_1 = 0$ - a moderately thin atmosphere with

$$\tau_{atm, v} = \tau_{\text{continuum}} + A e^{-\beta(v-v_0)^2}$$

apply the formal solution for thin atmosphere:

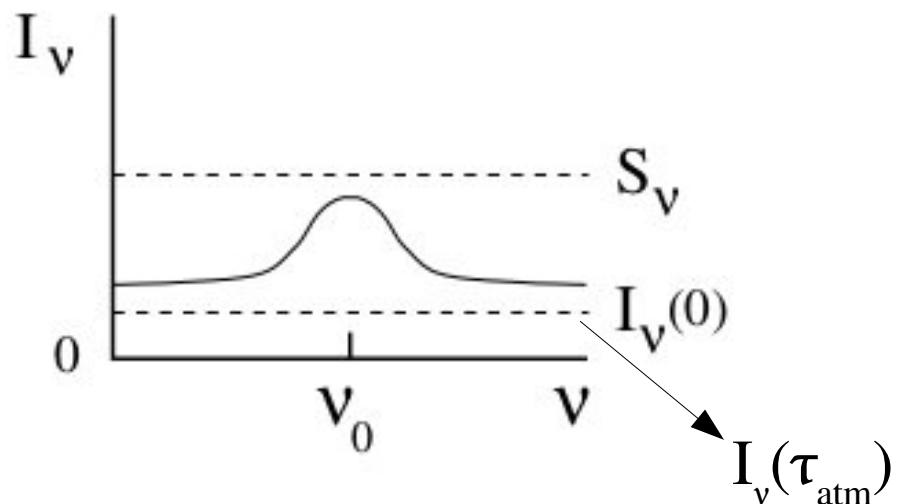
$$I_v(0, \mu=1) = I_v(\tau_{\text{atm}}) + [S_v - I_v(\tau_{\text{atm}})]\tau_{\text{atm}}$$



$$1.a \quad S_v > I_v(\tau_{\text{atm}}); \quad \tau_{\text{cont}} \ll 1; \quad \tau_{\text{line}} < 1$$

a thin continuum, weakly illuminated atmosphere

$$I_v(0, \mu=1) = I_v(\tau_{\text{atm}}) + [S_v - I_v(\tau_{\text{atm}})] \{\tau_{\text{cont}} + A \exp[-\beta(v-v_0)^2]\}$$



$$\begin{aligned} I_{\text{cont}}(0) &= I_v(\tau_{\text{atm}}) + [S_v - I_v(\tau_{\text{atm}})] \tau_{\text{cont}} \\ &\sim I_v(\tau_{\text{atm}}) \end{aligned}$$

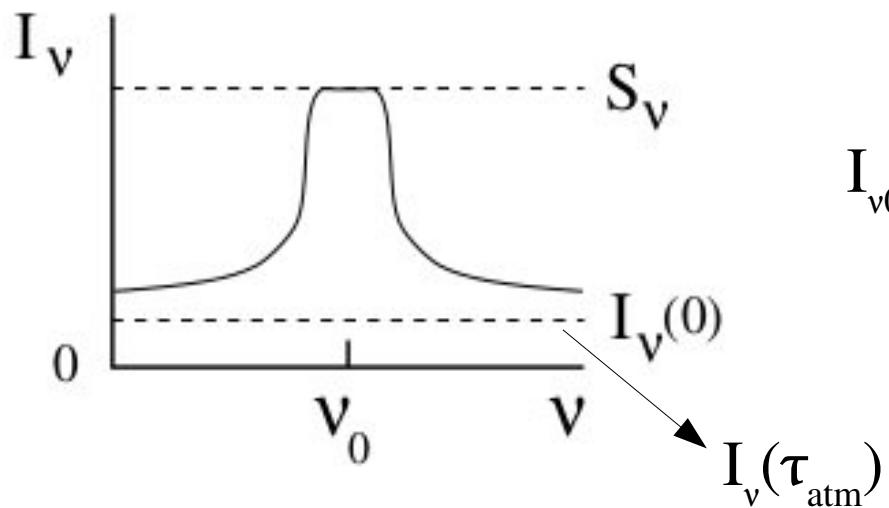
$$\begin{aligned} I_{v0}(0) &= I_v(\tau_{\text{atm}}) + \\ &[S_v - I_v(\tau_{\text{atm}})] (\tau_{\text{cont}} + A) \\ &\sim S_v A + (1-A) I_v(\tau_{\text{atm}}) \end{aligned}$$

with $A < 1$

$$1.b \quad S_v > I_v(\tau_{atm}); \quad \tau_{cont} \ll 1; \quad \tau_{line} = 1$$

a continuum thin, line thick, weakly illuminated atmosphere

$$I_v(0, \mu=1) = I_v(\tau_{atm}) + [S_v - I_v(\tau_{atm})] \{ \tau_{cont} + \exp[-\beta(v-v_0)^2] \}$$



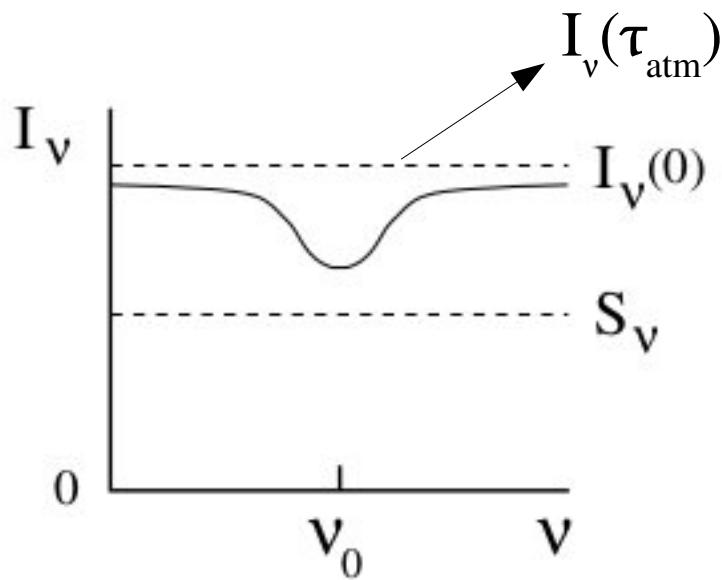
$$\begin{aligned} I_{cont}(0) &= I_v(\tau_{atm}) + [S_v - I_v(\tau_{atm})] \tau_{cont} \\ &\sim I_v(\tau_{atm}) \end{aligned}$$

$$\begin{aligned} I_{v0}(0) &= I_v(\tau_{atm}) + [S_v - I_v(\tau_{atm})] (\tau_{cont} + 1) \\ &\sim S_v \end{aligned}$$

$$2.a \quad I_v(\tau_{\text{atm}}) > S_v; \quad \tau_{\text{cont}} \ll 1; \quad \tau_{\text{line}} < 1$$

a continuum thin, line thin, strongly illuminated atmosphere

$$I_v(0, \mu=1) = I_v(\tau_{\text{atm}}) + [S_v - I_v(\tau_{\text{atm}})]\{\tau_{\text{cont}} + A \exp[-\beta(v-v_0)^2]\}$$



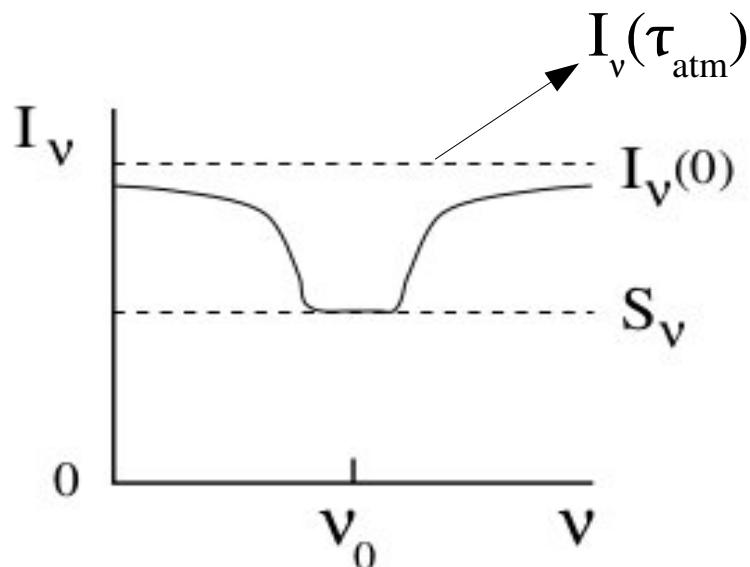
$$I_{\text{cont}}(0) = I_v(\tau_{\text{atm}}) + [S_v - I_v(\tau_{\text{atm}})] \tau_{\text{cont}}$$

$$I_{v0}(0) = I_v(\tau_{\text{atm}}) + [S_v - I_v(\tau_{\text{atm}})] (\tau_{\text{cont}} + A)$$

$$2.b \quad I_v(\tau_{\text{atm}}) > S_v; \quad \tau_{\text{cont}} \ll 1; \quad \tau_{\text{line}} = 1$$

a continuum thin, line thick, strongly illuminated atmosphere

$$I_v(0, \mu=1) = I_v(\tau_{\text{atm}}) + [S_v - I_v(\tau_{\text{atm}})]\{\tau_{\text{cont}} + \exp[-\beta(v-v_0)^2]\}$$



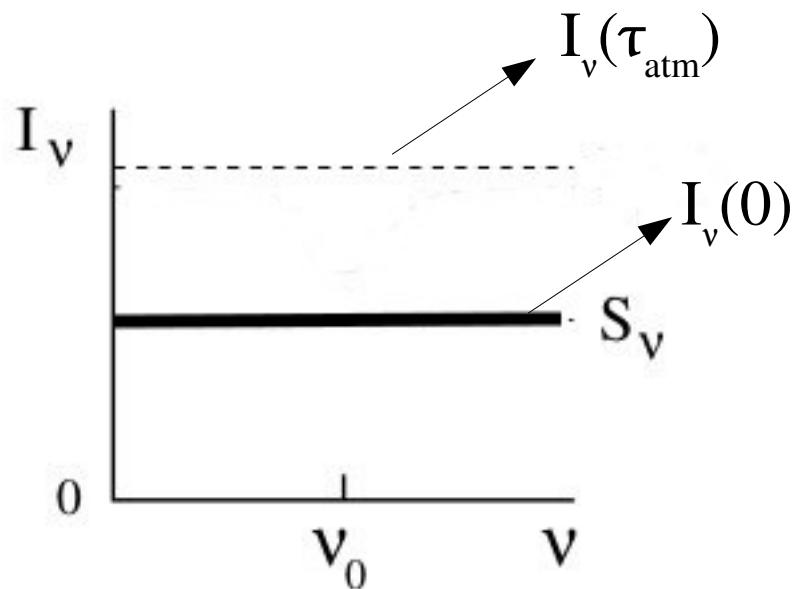
$$I_{\text{cont}}(0) = I_v(\tau_{\text{atm}}) + [S_v - I_v(\tau_{\text{atm}})] \tau_{\text{cont}} \\ \sim I_v(\tau_{\text{atm}})$$

$$I_{v0}(0) = I_v(\tau_{\text{atm}}) + [S_v - I_v(\tau_{\text{atm}})] (\tau_{\text{cont}} + 1) \\ \sim S_v$$

$$3.a \quad I_v(\tau_{\text{atm}}) > S_v; \quad \tau_{\text{cont}} = 1;$$

a continuum thick, small A, strongly illuminated atmosphere

$$I_v(0, \mu=1) = I_v(\tau_{\text{atm}}) + [S_v - I_v(\tau_{\text{atm}})]\{\tau_{\text{cont}} + A \exp[-\beta(v-v_0)^2]\}$$



$$I_{\text{cont}}(0) = I_v(\tau_{\text{atm}}) + [S_v - I_v(\tau_{\text{atm}})] \tau_{\text{cont}}$$
$$\sim S_v$$

$$I_{v0}(0) \sim S_v$$

ii. Semi-infinite atmosphere with a linear source function

$$S_v = a + b\tau_v$$

$$I(0, \mu) = \frac{1}{\mu} \int_0^{\infty} (a + b\tau') e^{-\frac{\tau'}{\mu}} d\tau'$$

$$I(0, \mu) = a + b\mu = S_v(\tau_v = \mu)$$

(Eddington-Barbier relation)

The evaluation of $I_v(0)$ at a given “ μ ” provides a good approximation to the actual value of the source function at depth $\tau_v = \mu$.